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Chapter 4. Global Positioning System

- *How does the Global Positioning System [GPS] work?*
- *How accurately can I locate myself on Earth with the GPS?*
- *Does the GPS operate properly if I "turn off general relativity"?*
- *Why are so many people willing to pay for GPS devices?*

CHAPTER 4
Global Positioning System

Edmund Bertschinger & Edwin F. Taylor *

> *There is no better illustration of the unpredictable payback of fundamental science than the story of Albert Einstein and the Global Positioning System [GPS] . . . the next time your plane approaches an airport in bad weather, and you just happen to be wondering "what good is basic science," think about Einstein and the GPS tracker in the cockpit, guiding you to a safe landing.*
>
> —Clifford Will

4.1 ■ OPERATION OF THE GLOBAL POSITIONING SYSTEM

Relativistic effects of altitude and speed on clock rates

General relativity: Crucial to GPS functioning.

Do you think that general relativity concerns only events far from common experience? Think again! Your hand-held Global Positioning System (GPS) receiver "listens" to overhead satellites and tells you where you are anywhere on Earth. In this chapter you show that the operation of the GPS system depends fundamentally on general relativity.

GPS satellite system

The Global Positioning System includes a network of 24 operating satellites, in circular orbits around Earth with orbital period of 12 hours, distributed in six orbital planes equally spaced in angle (Figure 4.1). Each satellite carries an operating atomic clock (along with several backup clocks) and emits a timed signal that also codes the satellite's location. By analyzing signals from at least four of these satellites (Box 4.1), a receiver in your hand on Earth displays your own location (latitude, longitude, and altitude). Consumer receivers provide a horizontal position accurate to approximately 5 meters. Among its almost endless applications, GPS guides your driving, flying, hiking, exploring, rescuing, mapmaking, and locating your dog.

General relativity: Position and motion effects

The timing accuracy required by the GPS is so great that general relativistic effects are central to its performance: First, different altitudes affect the rate of a stationary clock. Second, relative motion between the satellite clock and the clock on Earth's surface affect the results.

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*Draft of Second Edition of *Exploring Black Holes: Introduction to General Relativity*

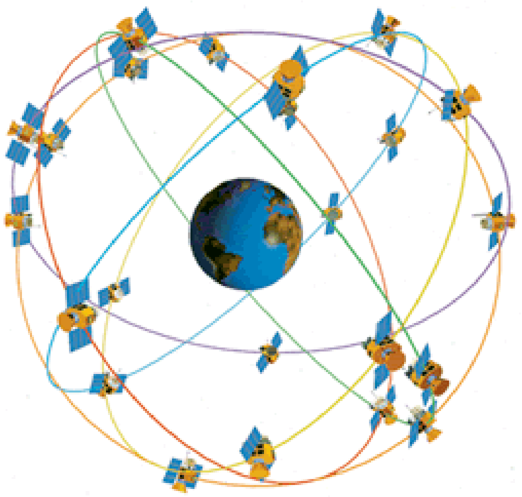

FIGURE 4.1 Schematic plot of GPS satellites in 12-hour orbits around Earth, not to scale.

Many **Queries**

This chapter makes heavy use of **Queries**, in-text questions that ask you to fill out steps of derivations and to apply results to predict observations. Typically a Query contains several related questions. Answer the queries in order, or as assigned to you, or skip to those that interest you. The first Query appears on page 4-6. Because there are so many Queries in this chapter, we omit end-of-chapter exercises.

4.2 ■ STATIONARY CLOCKS

Warping of time at different altitudes.

Simplest case: clock on tower and no Earth rotation.

The Global Positioning System depends on the reception by a receiver on Earth's surface of microwave signals from multiple overhead satellites. Begin with the simplest possible case: Earth does not rotate and the higher clock is not in a satellite but rather sits on the top of a tower at **H**igher map radius r_H. The satellite communicates with us on Earth, at **L**ower map radius r_L. Calculate the radially-downward map velocity of microwaves that move with light speed from the top to the bottom of the tower. For these conditions, $d\phi = 0$ and for light, $d\tau = 0$. Then the Schwarzschild metric, equation (3.5), yields the following radial map velocity:

$$\frac{dr}{dt} = -\left(1 - \frac{2M}{r}\right) \qquad \text{(light moving radially downward)} \qquad (4.1)$$

Map speed of light $\neq 1$.

Is equation (4.1) a surprise? For the first time in our study of relativity, calculated light speed differs from one meter of distance per meter of time. Chapters 12 through 16 examine light propagation in detail. The following derivation makes use of the fundamental difference between map time in

Box 4.1. Practical Operation of the Global Positioning System

The goal of the Global Positioning System (GPS) is to determine your position on Earth in three dimensions: east-west, north-south, and vertical—longitude, latitude, and altitude. Signals from three overhead satellites provide this information. Each satellite emits a signal that encodes time of emission and the satellite's position at that time, this position being continually calculated using data uploaded from control stations on the ground. Your hand-receiver clock times the reception of each signal, then subtracts the emission time to determine the time lapse and hence how far the signal has traveled at the speed of light. This is the distance the satellite was from your position when it emitted the signal. In effect, the receiver constructs three spheres from these distances, one sphere centered on the emission point of each satellite. By simple triangulation, you stand at a point where the three spheres intersect.

Of course there is a wrinkle: The clock in your hand-held receiver is not nearly so accurate as the atomic clock carried in a satellite. For this reason, the signal from a fourth overhead satellite is employed to correct the clock in your receiver. This fourth signal enables your hand receiver to process GPS signals as though it contained an atomic clock.

Signals exchanged between atomic clocks at different altitudes and moving at different speeds are subject to general relativistic effects. Neglect these effects and the GPS is useless (Box 4.3).

arbitrary global coordinates (not measured directly by anyone) and two wristwatch time lapses: $d\tau_{\mathrm{H}}$ at the tower-top map radius r_{H} and $d\tau_{\mathrm{L}}$ at the lower Earth map radius r_{L}. As usual, the metric converts from map time (on its right side) to wristwatch time (on its left side), which we can choose to be the time lapse between ticks on any clock.

Tower clock emits two downward flashes.

The clock at the top of the tower emits two flashes radially downward (emission events A and B) differentially close together in map time, dt_{AB}. For the stationary tower clock, $dr = 0$ and $d\phi = 0$, the metric tells us the corresponding wristwatch time lapse $d\tau_{\mathrm{H}}$ recorded on the tower clock:

$$d\tau_{\mathrm{H}} = \left(1 - \frac{2M}{r_{\mathrm{H}}}\right)^{1/2} dt_{\mathrm{AB}} \qquad (d\phi = 0,\ dr = 0) \qquad (4.2)$$

Map time lapse between flashes same at all radii.

Figure 4.2 traces the radially-downward worldlines of the two flashes emitted by the tower clock at events A and B. These flashes are received by the Earth clock at events C and D with map time separation dt_{CD} (reception events C and D). Equation (4.1) tells us that these worldlines have identical slopes (the radial map speed of light has the same value) at every intermediate value r of map radius. As a result, the two worldlines are parallel at every radius on the global map spacetime diagram, so the map time between them remains at the initial value dt_{AB}. The two flashes arrive at the ground with the same difference in *map* time between them.

$$dt_{\mathrm{CD}} = dt_{\mathrm{AB}} \qquad (4.3)$$

The clock on the surface of the (non-rotating!) Earth is also at a fixed, single value of the radius r_{L}. Therefore its wristwatch time on the ground between the reception of events is similarly given by (4.2):

$$d\tau_{\mathrm{L}} = \left(1 - \frac{2M}{r_{\mathrm{L}}}\right)^{1/2} dt_{\mathrm{CD}} = \left(1 - \frac{2M}{r_{\mathrm{L}}}\right)^{1/2} dt_{\mathrm{AB}} \qquad (4.4)$$

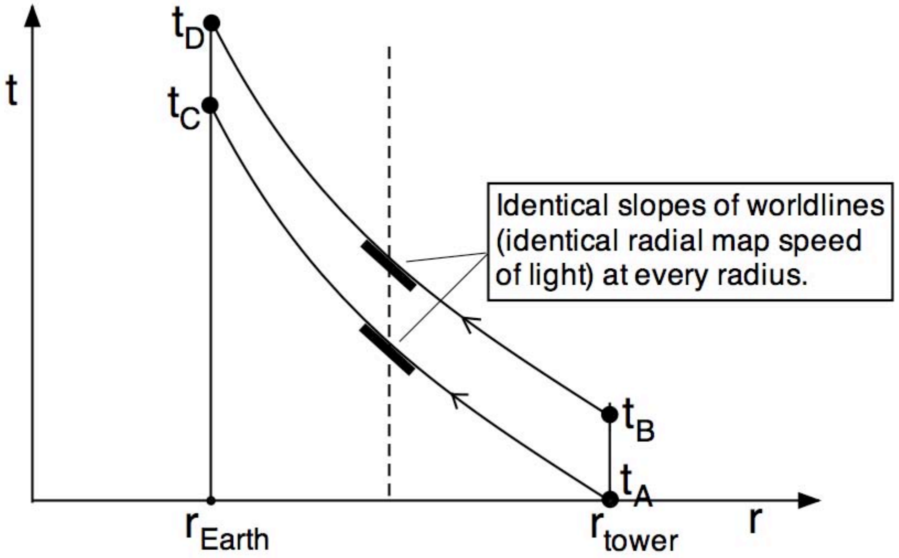

FIGURE 4.2 Schematic plot in Schwarzschild map coordinates t, r of worldlines of two sequential flashes moving downward from the top of a tower. According to equation (4.1) the map velocity depends only on map radius r. As a result, the *map* time difference between reception of the flashes is identical to the *map* time difference between emission of the flashes. However, the *wristwatch* time between the reception events C and D, measured by the bottom observer, is different from the *wristwatch* time between emission events A and B, measured by the top observer. The figure greatly exaggerates the variation of radial map light velocity with radius.

The final step in (4.4) comes from (4.3). Equations (4.2) and (4.4) gives us the relation between wristwatch time lapses of stationary locks at higher map radius and lower map radius:

$$\frac{d\tau_{\rm H}}{d\tau_{\rm L}} = \left(\frac{1-\dfrac{2M}{r_{\rm H}}}{1-\dfrac{2M}{r_{\rm L}}}\right)^{1/2} \qquad \text{(stationary clocks)} \qquad (4.5)$$

Clock time lapse between flashes is different at different radii.

The lapse in Schwarzschild *map* time dt between flashes is the same at the locations of both clocks, but the *wristwatch* time is different as recorded on these different clocks. Indeed, $r_{\rm L} < r_{\rm H}$, so equation (4.5) tells us that $d\tau_{\rm H} > d\tau_{\rm L}$; the lapse of wristwatch time on the higher clock is greater than the lapse of wristwatch time on the lower clock. The somewhat sloppy everyday expression for this inequality is, "A clock at a greater altitude runs faster," in spite of the fact that each Earth and tower observer thinks (correctly!) that his clock runs at its normal rate (Box 4.2).

Gravitational red and blue shifts

The wristwatch time lapse $d\tau_{\rm H}$ on the higher clock can be envisioned as the measured period $T_{\rm H}$ of a sinusoidal signal emitted from the top of the tower. The measured period T_{L} of the signal as it reaches Earth's surface is therefore observed to be smaller. Frequency is inversely proportional to period, so the observed frequency of the signal increases as it descends. This is called the **gravitational blue shift**. In contrast, for a signal rising from Earth's surface to be observed at the top of the tower, the period increases and the frequency decreases, an effect labeled **gravitational red shift**.

4.3 ■ APPROXIMATIONS

How small is small?

GR effects: small but crucial to GPS.

The general relativistic effects we study are small. How small? Small compared to what? When *can* we use approximations to general relativistic expressions? And when we do, which approximations are good enough? These questions are so central to the analysis of the GPS that it is useful to begin with a rough estimate of the expected effects, not worrying initially about the crudeness of this approximation.

Assume that our stationary tower—standing on a non-rotating Earth—extends to the height of the GPS satellite and that the satellite rests without moving on the top of that tower ($v = 0$). Write (4.5) in the form

$$\frac{d\tau_{\mathrm{H}}}{d\tau_{\mathrm{L}}} = \left(1 - \frac{2M}{r_{\mathrm{H}}}\right)^{1/2} \left(1 - \frac{2M}{r_{\mathrm{L}}}\right)^{-1/2} \qquad \text{(stationary clocks)} \qquad (4.6)$$

You show in Query 4.7 that the radius of a 12-hour circular orbit is about 26.6×10^6 meters from Earth's center. Inside the front cover are values for the radius and mass of Earth. We now make use of an approximation also written inside the front cover:

$$(1+\epsilon)^n \approx 1 + n\epsilon + O(\epsilon^2) \quad \text{provided} \quad |\epsilon| \ll 1 \quad \text{and} \quad |n\epsilon| \ll 1 \qquad (4.7)$$

Our approximations are "to first order," that is, we neglect the correction term $O(\epsilon^2)$, which means "terms of second (and higher) order in ϵ."

QUERY 4.1. Clock rate difference due to difference in altitude.

Apply approximation (4.7) in first order to the two parenthetical expressions on the right side of equation (4.6). Multiply out the result to show that to first order:

$$\frac{d\tau_{\mathrm{H}}}{d\tau_{\mathrm{L}}} \approx 1 - \frac{M}{r_{\mathrm{H}}} + \frac{M}{r_{\mathrm{L}}} \qquad (v = 0 \text{ and nonrotating Earth}) \qquad (4.8)$$

Verify that approximate values of both M/r_{Earth} and $M/r_{\mathrm{satellite}}$ satisfy the criteria for approximation (4.7) that leads to the result (4.8).

QUERY 4.2. Numerical approximation, stationary clocks.
In the following equation, b stands for the sum of two terms added to the number one on the right side of equation (4.8). Substitute numbers into equation (4.8) and find the numerical value of b:

$$\frac{d\tau_{\rm H}}{d\tau_{\rm L}} \approx 1 + b \qquad (v = 0 \text{ and nonrotating Earth}) \tag{4.9}$$

Small fractional differences in clock rates affect GPS operation.

The numerical value of b in equation (4.9) gives us an estimate of the fractional difference in rates of signals between stationary clocks at the position of the satellite and at Earth's surface. Is this fractional difference negligible or important to the operation of the GPS? Suppose the timing of a satellite clock is off by one nanosecond (10^{-9} second). In one nanosecond a light signal (or microwave pulse) propagates approximately 30 centimeters. So a difference of, say, hundreds of nanoseconds will render GPS results inaccurate if we need a location precision of ten meters or so.

QUERY 4.3. Synchronization discrepancy after one day.
As long as Earth and satellite clocks do not move, the time increments in equation (4.9) can be as long as we want, leading to the equation

$$\tau_{\rm H} \approx (1 + b)\, \tau_{\rm L} \qquad (v = 0 \text{ and nonrotating Earth}) \tag{4.10}$$

There are approximately 86 400 seconds in one day. (Since this is approximate, which clock records it does not matter.) To an accuracy of one significant digit, the satellite clock and Earth clock go out of synchronism by about 50 000 nanoseconds per day due to their difference in altitude alone. Find the correct value to three-digit accuracy.

The satellite clock will "run fast" by something like 50 000 nanoseconds per day compared with the clock on Earth's surface due to position effects alone. Clearly general relativity is needed for useful operation of the Global Positioning System, even though the *fractional* difference between clock rates at the two locations (at least the fraction due to difference in radius) is small.

Velocity effects opposite to altitude effects.

In addition to the effect of altitude, we must include the effect due to relative motion between satellite and Earth observer. Which way will these effects influence the discrepancy due to altitude introduced by general relativity: to increase it or decrease it? The satellite clock now moves with respect to a string of Earth clocks. Special relativity tells us (in an imprecise summary): "Speeding clocks run slow." Therefore we expect the effect of motion to *reduce* the amount by which the satellite clock runs fast compared to the Earth clock. In brief, when velocity effects are taken into account, we expect the satellite clock to run faster than the Earth clock by *less* than the estimated 50 000 nanoseconds per day. In Query 4.8, you check your final result against this prediction.

4.4 ■ MOVING CLOCKS

Relative velocity changes relative clock rates

Add rotating Earth and orbiting satellite

Now we take account of the effects of motion on the relative rates of Earth clock and satellite clock. Think of the Earth clock as fixed at the equator, so that it moves in a circle as the Earth turns. The satellite clock also circles the Earth, but in its own independent circular orbit, so $dr = 0$ for each clock. In each case $d\tau$ is the wristwatch time between ticks, the time recorded by a given clock. Set $dr = 0$ in the Schwarzschild metric and divide through by the square of the Schwarzschild map time dt^2 to obtain, for either clock in its orbit of radius r,

$$\left(\frac{d\tau}{dt}\right)^2 = \left(1 - \frac{2M}{r}\right) - r^2\left(\frac{d\phi}{dt}\right)^2 = \left(1 - \frac{2M}{r}\right) - v^2 \qquad (dr = 0) \qquad (4.11)$$

Here $d\tau$ is the wristwatch time between ticks of either clock and $v = rd\phi/dt$ is the instantaneous tangential map velocity of that clock moving at its radius r.

QUERY 4.4. Clock rate correction formula.
First apply equation (4.11) to the satellite clock, then apply (4.11) to the Earth clock. Divide the two sides of the satellite equation by the corresponding sides of the Earth equation. Take the square root of both sides of the result. For both numerator and denominator in the resulting equation, use the approximation (4.7). In the numerator, set

$$\epsilon_{\mathrm{H}} = -\frac{2M}{r_{\mathrm{H}}} - v_{\mathrm{H}}^2 \qquad \text{(subscript H means satellite)} \qquad (4.12)$$

Now do the same for the denominator. In the denominator the formula for ϵ_{L} is the same as that for ϵ_{H}, but with L for "lower" as subscripts. Carry out an analysis similar to that in Query 4.1 to retain only the dominant terms. Show that the result is

$$\boxed{\frac{d\tau_{\mathrm{H}}}{d\tau_{\mathrm{L}}} \approx 1 - \frac{M}{r_{\mathrm{H}}} - \frac{v_{\mathrm{H}}^2}{2} + \frac{M}{r_{\mathrm{L}}} + \frac{v_{\mathrm{L}}^2}{2} \qquad \text{(satellite directly overhead)} \qquad (4.13)}$$

Newton orbits good enough for GPS analysis.

Now we need numerical values for the quantities on the right side of (4.13). Chapter 9 derives the velocity of a satellite in circular orbit according to general relativity. After completing that chapter you can verify that the following Newtonian description of the radius of any orbit and the satellite speed in that orbit provides orbital descriptions sufficiently accurate for our analysis of the Global Positioning System.

QUERY 4.5. Speed of a clock on the equator

Earth's center is in free fall as Earth orbits the Sun. The Earth also rotates on its axis, completing one full rotation with respect to the distant stars in what is called a **sidereal day**, which is 86 164.1 seconds long. (Even when we require 6-digit accuracy, which of our clocks measures this time does not matter.) With respect to Earth's center, what is the speed v of a clock at rest on Earth's surface at the equator? Use Newtonian "universal time" t. Express your answer as a fraction of the speed of light.

What is the value of the speed v of the satellite? Newton tells us that the acceleration of a satellite in a circular orbit is directed toward the center and has the magnitude v^2/r, where v is measured in conventional units, such as meters per second. The satellite's mass m multiplied by this acceleration must equal Newton's gravitational force exerted by Earth:

$$\frac{GmM}{r^2} = \frac{mv^2}{r} \qquad \text{(Newton, conventional units)} \qquad (4.14)$$

Newtonian orbit analysis

Equation (4.14) provides one relation between the speed of the satellite and the radius of its circular orbit. A second relation connects satellite speed and orbit radius to the period of revolution. This period T is 12 hours for GPS satellites:

$$v = \frac{2\pi r}{T} \qquad \text{(Newton, conventional units)} \qquad (4.15)$$

QUERY 4.6. Units in meters

Convert equations (4.14) and (4.15) to units in meters: Earth mass M and satellite orbital period T to meters; satellite speed v to the unitless fraction of the speed of light. Then eliminate the radius r between these two equations to find an expression for v in terms of M and T and numerical constants.

Box 4.2. OPINION: Slogans and Observations

“Moving clocks run slow.” Special relativity gives us this useful slogan, a slogan that follows us into general relativity, which adds a second useful slogan, “Clocks higher in a gravitational field run faster.” What do these slogans *mean*?

First of all, we need to specify “faster or slower” with respect to what? More precisely, special relativity says, “An observer measures a clock moving past him to run slower than a clock at rest next to him.”' General relativity adds, “An observer at lower altitude in a gravitational field may interpret signals he receives from a clock above him to mean that the higher clock runs faster than his own clock.” The GPS verifies these slogans and demonstrates their usefulness.

But there is a deeper issue: When you ride in a spaceship speeding past your friend, your wristwatch does not run slow *for you*; it ticks at it’s accustomed pace compared, for example, with your pulse—or your aging! Similarly, when you mount a ladder vertically away from your friend in a gravitational field, you notice no change in your wristwatch rate; for you it does not speed up as you gain altitude.

So does a clock *really* slow down as it moves faster? Does a clock *really* speed up as it rises in a gravitational field? Welcome to relativity: *Observations are relative; they depend on the observer!*

Physics does not say what *reality* is; it formulates theories to explain observations and to predict new ones.

4.5 ■ THE FINAL RECKONING

Effects of altitude AND relative speed on clock rates

Comment 4.1. Relative Motion Leads to Doppler Shift

Is the GPS satellite approaching the ground receiver or receding from it? If the satellite approaches, the receiver detects a Doppler increase in frequency of the incoming signal. In contrast, it detects a Doppler downshift in frequency when the satellite recedes from the Earth receiver. In this chapter we carry out calculations for the instant at which the orbiting satellite is positioned directly above the Earth receiver. In this case the change in the detected signal frequency is due to the relative tangential motion between the two clocks and to their different altitudes. For other relative motions of satellite and receiver, the computer in the receiver calculates the anticipated Doppler shift and adjusts the derived location accordingly.

QUERY 4.7. Satellite radius and speed, according to Newton.

Find the numerical value of the speed v (as a fraction of the speed of light) for a satellite in a 12-hour circular orbit. Find the numerical value of the radius r for this orbit—according to Newton and Euclid.

Now we have numerical values for all the terms in equation (4.13) and can estimate the difference in rate between satellite and Earth clocks.

QUERY 4.8. Clock rate correction, numerical
Substitute values for the various quantities in (4.13). Show that the satellite clock appears to run faster than the Earth clock by approximately 38 700 nanoseconds per day.

Section 4.3 described the difference in clock rates due only to difference in altitude. We predicted at the end of Section 4.3 that the full general relativistic treatment would lead to a *smaller* difference in clock rates than reckoned for the altitude effect alone. Your result for Query 4.8 verifies this prediction.

Before launch, GPS satellite clocks are set to run at a rate of 38 700 nanoseconds per day *slower* than identical Earth clocks next to them, clocks that will remain on Earth's surface. *Result:* When the satellite clock passes overhead, the increased frequency (blue shift) of its signal received on Earth synchronizes with the Earth clock (Box 4.3).

Gravitational shifts: no single identifiable cause

An historical aside: Carroll O. Alley, a consultant to the original GPS project, had a hard time convincing the designers not to apply *twice* the correction given in (4.13): first to account for the difference in clock rates at different altitudes and second to allow for the gravitational blue shift in frequency for the signal sent downward from satellite to Earth. There is only one correction; moreover there is no way to identify uniquely the "cause" of this correction. Listen to what Clifford Will says about the difference in rates between one clock emitting a signal from the top of a tower and a second identical clock receiving the signal on the ground:

> **Comment 4.2. What is the CAUSE of different clock rates?**
> "A question that is often asked is, Do the intrinsic rates of the emitter and receiver or of the clock change, or is it the light signal that changes frequency during its flight? The answer is that it doesn't matter. Both descriptions are physically equivalent. Put differently, there is no operational way to distinguish between the two descriptions. Suppose that we tried to check whether the emitter and the receiver agreed in their rates by bringing the emitter down from the tower and setting it beside the receiver. We would find that indeed they agree. Similarly, if we were to transport the receiver to the top of the tower and set it beside the emitter, we would find that they also agree. But to get a gravitational red shift, we must separate the clocks in height; therefore, we must connect them by a signal that traverses the distance between them. But this makes it impossible to determine unambiguously whether the shift is due to the clocks or to the signal. The observable phenomenon is unambiguous: the received signal is blue shifted. To ask for more is to ask questions without observational meaning. This is a key aspect of relativity, indeed of much of modern physics: we focus only on

Box 4.3. General Relativity On/Off Switch

Launching the Global Positioning System was an immense military and civilian effort. Most participants were not skilled in general relativity and, indeed, wondered if the academic advisors were right about this strange theory. As one later publication put it:

> *There was considerable uncertainty among Air Force and contractor personnel designing and building the system whether these effects were being correctly handled, and even, on the part of some, whether the effects were real.*

The GPS prototype satellite called Navigation Technological Satellite 2 (NTS-2) was launched into a near-12-hour circular orbit on June 23, 1977, with its single atomic clock initially set (on Earth) to run at the same rate as Earth clocks. However, it had a general relativity on/off switch, leading to two possible modes of operation. In the first mode, with the switch set to ”off”, the satellite clock was simply left to run at the rate at which it had been set on Earth. It ran in this condition for 20 days. The satellite clock drifted in time, compared with Earth clocks, at the rate predicted by general relativity, “well within the accuracy capabilities of the orbiting clock.”

The NTS-2 satellite validated the general relativity results, so the on/off switch was turned to “on.” This changed the satellite clock rate to a pre-arranged 38 700 nanoseconds per day slower than that of the Earth clock, also set before launch when the two clocks were next to each other on Earth. Then the gravitational blue shift of the signal from an orbiting overhead satellite raised the frequency of the signal received on Earth to that of the Earth clocks. Now every GPS satellite goes into orbit with general relativity built into its design and construction.

> observable, operationally defined quantities, and avoid unanswerable questions.”

The ambiguity described by Clifford Will appears in our treatment of clock rates at different radii. Box 3.6 started with equal wristwatch times: $d\tau_{\rm H} = d\tau_{\rm L} = 1$ second, and derived different map times: $dt_{\rm H} \neq dt_{\rm L}$. In contrast, Section 4.2 notes that the map time lapse between two radially-directed signals does not change as the signals travel between locations: $dt_{\rm H} = dt_{\rm L}$ and from this derives a difference in clock time lapses: $d\tau_{\rm H} \neq d\tau_{\rm L}$. Clifford Will tells us that both methods lead to the same numerical result.

TWO COMMENTS

Comment 4.3. Newtonian orbit radius OK.
The approximate analysis in this chapter assumes that the radius $r_{\rm H}$ of the circular orbit of the satellite and the velocity v of the satellite in that orbit are both computed accurately enough using Newtonian mechanics. In contrast, Chapter 9 carries out the Schwarzschild analysis of circular orbits. When you have completed that chapter, you will be able to show that Newtonian values of orbit radius and velocity are sufficiently accurate to describe the orbit of a GPS satellite.

Comment 4.4. Little latitude effect.
Our analysis assumes that the speed of the Earth clock is that of a clock fixed to the ground at the equator. One might expect that this speed-dependent correction would take on different values for an Earth clock fixed to the ground at different latitudes north or south of the

Box 4.4. Shoes that Keep Track

"People with dementia or Alzheimer's who wander off and get lost are a frightening, frustrating problem for family caregivers and nursing facilities. Those who are not located quickly risk dehydration, exposure or injury. But here's one new solution: a locator shoe with a built-in Global Positioning System device that makes it easier to track down its wearers. The shoes, manufactured by GTX Corp., look like typical walking shoes but have a miniature GPS unit implanted in the heel. They sell for about $300. The shoe works by allowing caregivers or family members to set a perimeter, called a "geo-fence," that allows wearers to move freely around a specific area. When they stray beyond the perimeter, a Google Maps message pops up on a computer or phone to alert caregivers."

—Candy Sagon

AARP Bulletin, December 2011, page 8

equator, going to zero at the poles where there is no motion of the Earth clock due to rotation of Earth. In practice there is negligible latitude effect because Earth is not spherical; it bulges a bit at the equator due to its rotation, like a squashed balloon. The smaller radius at the poles increases the M/r_{L} term in (4.13) by roughly the same amount that the velocity term decreases. The outcome is that our calculation for the equator applies quite well to all latitudes.

4.6 ■ APPLICATIONS OF THE GLOBAL POSITIONING SYSTEM

What are the applications of the wristwatch?

Since the first edition of this book in the year 2000, applications of the Global Positioning System have exploded. To ask how the GPS system is used today is like asking about applications of the wristwatch or the telephone. At one end, geologists measure the millimeters-per-year motion of the continents (motion with respect to what?); at the other end, the GPS corrals Alzheimer patients (Box 4.4). How is the GPS used? Look around you!

QUERY 4.9. Aging on the International Space Station.

The International Space Station (ISS) circles the Earth at an altitude of approximately 350 kilometers, or at a radius of 6.73×10^6 meters at an orbital speed of 7 707 meters per second. An astronaut lives on the ISS for one year. When she returns to Earth's surface, how much younger (or older?) is she than her twin sister who stayed on Earth?

4.7 ■ REFERENCES

Initial quote from “A Tale of Einstein, Congress and Safe Landings” by Clifford Will, an online posting by the Editorial Services, Washington University, St. Louis.

Box 4.3 quote about general relativity on/off switch: Carroll O. Alley, “Proper Time Experiments in Gravitational Fields With Atomic Clocks, Aircraft, and Laser Light Pulses,” In *Quantum Optics, Experimental Gravity, and Measurement Theory,* edited by Pierre Meystre and Marlan O. Scully, Plenum Publishing, New York, 1983, pages 421-424.

Quote about clocks on the ground and on a tower: Clifford M. Will, *Was Einstein Right? Putting General Relativity to the Test,* Second Edition, Basic Books/Perseus Group, New York, 1993, pages 49-50.