GENERAL RELATIVITY BRIEFING

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GENERAL RELATIVITY BRIEFING

For further information on these terms, consult the index.

EVENT
An occurrence that we can locate at a point in space at an instant of time, such as your birth. Events are the elemental nails on which physics hangs. An event exists independent of any method we use to locate it.

SPACETIME
The arena in which events occur. Newton thought that time is universal, the same for all observers. Einstein realized that different observers typically measure different values of time between events, along with different values of their spatial distance. Einstein combined space and time to provide a spacetime measure of separation between events on which all observers agree: see "interval" and "metric" below.

SPACETIME REGION
A volume of space during a period of time, for example the vicinity of a black hole during one year.

GRAVITY
In Newtonian physics, a universal force arising from mass. For Newton, forces may be real (such as gravity) or fictitious (such as centrifugal force or Coriolis force) and we can correctly analyze gravity in a single global inertial coordinate system ("inertial" defined later). In general relativity, however, gravity is always a fictitious force which we can eliminate locally by changing to a local frame that is in free fall (a different free fall frame for each event).

CURVATURE
A measure of those properties of spacetime which prevent us from having a global inertial frame. Sources of curvature include mass-energy and pressure.

GENERAL RELATIVITY
A theory of curved spacetime and motion.

SINGULARITY
A spacetime location at which curvature is so extreme that general relativity fails.

PATCH
A spacetime region purposely limited in size and duration so that curvature does not noticeably affect a given measurement or observation. A patch can have uniform gravity (which generates no curvature). We can approximate curved spacetime to any desired accuracy with a set of patches, in the same way that we can cover a floor curving down toward a drain with "sufficiently small" flat tiles.

STONE
A free particle whose location at each instant is described by an event and whose mass warps spacetime too little to be measured. A stone has nonzero mass and moves slower than light with respect to every local inertial frame.

WORLDLINE
The path of a stone through spacetime. We can mark off a worldline with a chain of intermediate events, such as ticks of the stone's wristwatch. The **wristwatch time** along a worldline is the reading on the stone's wristwatch.

GLOBAL COORDINATE SYSTEM
Any system that assigns numbers ("coordinates") to an event in order to locate it in an extended spacetime region. General relativity frees us to use almost any global coordinate system in a given spacetime region. We usually analyze events that occur on a single spatial plane, for which three coordinates, such as (t, x, y) or (t, r, ϕ), suffice to locate an event.

FRAME
A patch onto which a local coordinate system has been installed. Local coordinates are limited to that single patch. Almost always we choose Cartesian coordinates, synchronized clocks, and assume or arrange that the frame is inertial, so special relativity is valid in the local frame.

INERTIAL or FREE-FALL FRAME
A frame in which special relativity is locally valid. In an inertial frame a free stone moves straight with constant speed, that is along a straight worldline. An inertial frame is available near any event except on singularities such as those inside black holes. In general relativity inertial frames are local; spacetime curvature precludes global inertial frames.

INTERVAL
Any one of three possible alternative directly-measured **spacetime separations** between events, analyzed here for two events recorded in an inertial frame. If a stone can travel directly from one event to the other, the time lapse on that stone's wristwatch is called the **wristwatch time** or **timelike interval**. If only a light flash is fast enough to travel directly from one event to the other, we say there is a **null interval** or **lightlike interval** between fhem. If nothing is fast enough to move between the two events, then there exists an alternative inertial frame such that the two events are simultaneous in that frame. The distance between the events measured along a ruler at rest in that alternative frame is called the **ruler distance** or **spacelike interval**.

GLOBAL METRIC
A function that satisfies Einstein's field equations, whose inputs are global coordinate differentials (such as dt, dr, $d\phi$) between an adjacent pair of events and whose output is the square of the interval between the events. The metric (plus the topology of the region) completely specifies local spacetime and gravitational effects within the global region in which it applies. The metric combines with the Principle of Maximal Aging (defined later) to predict the motion of stones and massless particles in curved spacetime.

FRAME METRIC
A metric expressed in local coordinates on a patch and valid only on that patch. A frame metric is *always* distinguished from a global metric by the use of: 1. the increment (Δ) instead of the differential (d); 2. an approximately equal sign ($\approx$); 3. subscripts on all local coordinate increments (such as Δt_{shell}) instead of the global differential (dt).

INVARIANT
A physical quantity that has the same value, whether measured or calculated, with respect to any possible frame or global coordinate system. Examples include the interval (wristwatch, lightlike, or ruler) between two infinitesimally close events, the wristwatch time along a worldline, the mass of a stone, and the mass of a center of gravitational attraction.

PRINCIPLE OF MAXIMAL AGING
A free stone follows the worldline of maximal wristwatch time (maximal aging) across any two adjoining local frames. We can completely tile spacetime with adjoining frames, so the stone knows how to move globally near a black hole or near Earth.

CONSTANT OF MOTION
A quantity describing the motion of stone or massless particle whose value does not change with time in the given global coordinate system. Constants of motion arise when all coefficients in the metric are independent of time or independent of one spatial coordinate. Energy and angular momentum are constants of motion for a free stone near a black hole.

METRIC ON A CURVED SURFACE *VS.* METRIC IN CURVED SPACETIME

ON A STATIC CURVED SURFACE IN SPACE:
First, we measure directly the DISTANCE between nearby points using a ruler or tape measure—or timed round trip of laser or radar pulse. This direct measurement requires no coordinates. Second, we set up a SURFACE MAP, which is just a rule that assigns unique coordinates to each point on the surface. Aside from some simple preconditions of uniqueness and smoothness, map coordinates are totally arbitrary. Third, we ask for the relation between the directly-measured distance between two given nearby points and their coordinate differences. The result is a local MAP SCALE between the two points. Finally we generalize, seeking a GLOBAL METRIC: a function whose input is coordinate differences between a pair of nearby points anywhere on the surface and whose output is the squared distance between them.

The output of the metric is an INVARIANT: the directly-measured distance between two nearby points has the same value no matter what map coordinates we choose.

Almost everywhere on our surface we can treat as FLAT a sufficiently small surface patch containing any pair of nearby points. On this patch we may set up a Cartesian coordinate system (in which the Pythagorean theorem is valid) in order to find the invariant distance between points. We can choose to make every measurement on each pair of nearby points with respect to such a local flat frame.

The local flat metric derives from the global metric. The reverse is not true; the local flat metric tells us nothing about the curved surface from which is projected.

IN A REGION OF CURVED SPACETIME:
First, we measure directly the SPACETIME SEPARATION (*timelike*, *lightlike*, or *spacelike* interval) between nearby events. This direct measurement requires no coordinates. Second, we set up a SPACETIME MAP, which is just a rule that assigns unique coordinates to each event in the spacetime region. Aside from some simple preconditions of uniqueness and smoothness, map coordinates are totally arbitrary. Third, we ask for the relation between the directly-measured spacetime separation between two given nearby events and their coordinate differences. The result is a local MAP SCALE between the two events. Finally we generalize, seeking a GLOBAL METRIC: a function that satisfies Einstein's field equations whose input is coordinate differences between a pair of nearby events anywhere in the spacetime region and whose output is the squared spacetime separation between them.

The output of the metric is an INVARIANT: the directly-measured spacetime interval between two nearby events has the same value no matter what map coordinates we choose.

Almost everywhere in our spacetime region we can treat as FLAT a sufficiently small spacetime patch containing any pair of nearby events. On this patch we may set up a local inertial frame (in which special relativity is valid) using Cartesian coordinates and synchronized clocks. We choose to make every measurement and observation on each pair of nearby events with respect to such a local inertial frame.

The local frame metric derives from the global metric. The reverse is not true; the local inertial metric tells us nothing about the global metric from which it is projected.