Selected Physical and Astronomical Constants

[Inside front cover, left side]

Gravitational constant $G = 6.67 \times 10^{-11}$ meter^3/(kilogram-second^2)

EARTH

Mass of Earth	$M_{\text{Earth}} = 5.97 \times 10^{24}$ kilogram $= 4.44 \times 10^{-3}$ meter
Equatorial radius of Earth	$r_{\text{Earth}} = 6.38 \times 10^{6}$ meter
Gravitational acceleration on Earth	$g_{\text{Earth}} = 9.81$ meter/second^2 $= 1.09 \times 10^{-16}$ meter^{-1}
Mean distance of Earth from Sun ≡ 1 astronomical unit (AU)	$1\,\text{AU} = 1.50 \times 10^{11}$ meter
Mean speed of Earth in its orbit around Sun	$v_{\text{Earth}} = 2.98 \times 10^{4}$ meter/second

OUR SUN

Mass of the Sun	$M_{\text{Sun}} = 1.99 \times 10^{30}$ kilogram $= 1.48 \times 10^{3}$ meter
Radius of the Sun	$r_{\text{Sun}} = 6.96 \times 10^{8}$ meter

OUR UNIVERSE

Age of the Universe	$= (13.7 \pm 0.1) \times 10^{9}$ year $= 1.30 \times 10^{26}$ meter
Hubble constant now	$H_0 = 72 \pm 2$ (kilometer/second)/Megaparsec $= 7.37 \times 10^{-11}$ (light year) $^{-1}$ $= 7.78 \times 10^{-27}$ meter^{-1}

Conversion Factors

Speed of light in a vacuum (by definition): $c \equiv 2.99792458 \times 10^{8}$ meter/second

1 second $\approx 3.00 \times 10^{8}$ meter of light-travel time

1 meter of light-travel time $= 3.34 \times 10^{-9}$ second

1 year $= 3.16 \times 10^{7}$ second $= 9.46 \times 10^{15}$ meter of light-travel time

1 kilometer $= 0.621$ mile

1 kilogram $= 7.42 \times 10^{-28}$ meter of mass $= 5.61 \times 10^{32}$ electron-volt

1 parsec $= 3.26$ light year

Approximation

$(1+\epsilon)^n \approx 1 + n\epsilon + O\left(\epsilon^2\right)$ $\qquad (|\epsilon| \ll 1$ and $|n\epsilon| \ll 1)$

For time in unit of meter, see Chapter 1, Speeding
For mass in unit of meter, see Chapter 3, Curving

Can I see a black hole at all? If I can see it, what does a black hole look like? Does it *look* black? Where do black holes exist in the Universe? Does the black hole look different when I fall toward it? What does it *feel* like to fall into a black hole? Am I comfortable? Do I see the stars overhead as I fall into a black hole? If so, do these stars change position or color as I fall? How fast do I fall? Does my speed reach or exceed the speed of light? Once inside, can I receive messages and packages from my friends on the outside? Is it true that, once inside, I cannot send anything to my friends on the outside, not even a light signal? Why not? What prevents me from sending them messages? How long do I live once I fall into a black hole? Will I reach the center alive? If not, why do I die—in what way does my body stop working? Is my death quick and painless? What is the last thing I see? Can I see the crunch-point ahead of me? What happens to the mass of a black hole when it swallows me or swallows a stone? How does the orbit of a stone around a black hole differ from the orbit of a planet around our Sun? Newton says a planet stays in orbit because the Sun exerts a gravitational force on it. How does Einstein account for this orbit? If Newton and Einstein disagree, how do we decide between them? How close to a black hole can I move in a permanent orbit? Can I travel backward in time using a black hole? What are the upper and lower limits on the masses of stars, white dwarfs, neutron stars, and black holes? Which of these bodies require general relativity for its correct description? In what sense are space and time unified? Why do things fall in my everyday life on Earth? Does the term *relativity* mean that everything is relative? What does *curvature* mean? How can I *observe* curvature? How many different effects require curvature for their description? How does the Global Positioning System fail us if we ignore general relativity? How much does light change direction as it passes the Sun or a black hole? Does the amount of change in direction depend on the color of the light? How does an astronomical object focus light from a distant galaxy and what does the image of that distant galaxy look like? Can light go into a permanent orbit around a black hole? How fast can a black hole spin? Does a spinning black hole drag space around with it? What does "dragging space" *mean*; how can I observe it? Can I extract energy from a spinning black hole? What is a quasar? Do spinning black holes power quasars; if so, how? What are gravitational waves? What can gravitational waves tell us about the Universe that light cannot? How far away is the most distant galaxy that we can see? Is the Universe just a big black hole? How did the Universe begin? What is the Universe made of? What does the *Big Bang* mean and how do we know it happened? Did time and space exist before the Big Bang? Why does the Universe expand with time? What is the Universe expanding into? Why is the expansion of the Universe speeding up? Where is the center of the Universe, anyway? How will the Universe end—or will it go on forever? Will the Earth exist twenty billion years from now? Whether or not the Earth exits then, what will the heavens look like from the position of our solar system?

Curiosity, like coffee, is an acquired need. Just a titillation
at the beginning, it becomes with training a raging passion.

Nicholas S. Thompson

For a General Relativity Briefing, see inside the back cover.

EXPLORING BLACK HOLES

INTRODUCTION TO GENERAL RELATIVITY

Second Edition

Edwin F. Taylor

John Archibald Wheeler

Edmund Bertschinger

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It is not my purpose in this discussion to represent the general theory of relativity as a system that is as simple and as logical as possible, and with the minimum number of axioms; but my main object here is to develop this theory in such a way that the reader will feel that the path we have entered upon is psychologically the natural one, and that the underlying assumptions will seem to have the highest possible degree of security.

—Albert Einstein

[REFERENCE: Albert Einstein, "The Foundation of the General Theory of Relativity" in *The Principle of Relativity*, translated by W. Perrett and G. B. Jeffery, 1952 Dover Publications, ISBN 0486600815, page 118.]

USING THIS BOOK . . .

The purpose of this book is to help you answer your own questions about black holes, our Earth, and our Universe, questions such as those inside the front cover. Answers to these questions require a theory of curved spacetime such as Einstein's general relativity, the subject of this book.

Your major tool in exploring general relativity is the **metric**, the solution of Einstein's spacetime equations that tells you what time or distance you will measure between a pair of events near to one another. The metric helps us to describe local and gravitational effects within the global region in which it applies. We provide metrics for spacetime around nonspinning and spinning black holes, for gravitational waves, and for the Universe as a whole. The most advanced mathematics required to apply a metric is simple calculus.

A General Relativity Briefing inside the back cover describes central concepts of the subject; a graphic on the back cover summarizes our strategy for presenting it. Refer to these summaries often to track your learning. At the beginning you may not understand all of the focal ideas; by the end these summaries will remind you of the wealth of topics you have mastered.

We do not tell the human story behind the development of general relativity and cosmology. We know of no better source of this background, told by a participant, than *Black Holes and Time Warps: Einstein's Outrageous Legacy* by Kip S. Thorne. We consider Thorne's book to be a perfect supplement to our own.

The World Wide Web offers rich resources on relativity and its applications. Typically, web material changes more often than sequential printings of this book, so our publisher maintains a website (FUTURE URL HERE) where we post updated web addresses of supplementary material, references, and resources. Please send comments, corrections, and suggestions for both the book and the website to eftaylor@mit.edu.

John Archibald Wheeler (1911-2008) co-authored the first edition of *Exploring Black Holes*. Starting in the 1950s, Wheeler rescued general relativity from obscurity and was a leader in jump-starting the present golden age of gravitational physics. He was immensely inventive in research and teaching, for example adopting and publicizing the name *black hole*. Wheeler's professional philosophy was *radical conservatism*: Take fundamental physical laws seriously, then drive them to the limit! The black hole is a perfect structure on which to apply radical conservatism, and our goal in this second edition is to refine and apply Wheeler's insights to teach general relativity. Wheeler treated every person with courtesy, respect, and helpfulness—whatever their status—and heartened everyone he met with his inventiveness and enthusiastic optimism. Those he taught, mentored, supervised, and inspired populate many fields inside and outside of science. We dedicate the second edition of this book to the memory of our treasured friend and colleague, John Archibald Wheeler.

For some, this book is an introduction to general relativity; for others it applies powerful ideas already mastered elsewhere. We hope that many will follow Wheeler's lead and experience this book as a personal journey into an awesome world: science fiction with numbers. "Fiction" only because we have not yet traveled to the black hole; the numbers are real.

Edmund Bertschinger
Edwin F. Taylor
Spring 2010