RESOURCE LETTER ON VARIATIONAL MECHANICS

Outline proposal submitted to the Editorial Committee for Resource Letters
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(see action publications of these two & others at www.eftaylor.com/leastaction.html)
Other volunteers may participate.
Thanks to Bartley Cardon for advice on this proposal.
All entries in the following outline will be encrusted with references.

I. OVERVIEW
Importance and scope of the action principle.
Limitations of the action principle.
This Resource Letter limited (mostly) to variational mechanics.
Initial sections describe single particle motion for simplicity.
Which expression is action?
Least action vs. stationary action

II. WHICH TOOLS FOR WHICH TASKS?
A. Finding the worldline
Abbreviated action for fixed initial and final *locations*.
(assumes conservation of energy)
Action for fixed initial and final *events*.
(yields conservation of energy)
Direct methods for finding worldline.

B. Finding the next step in motion
Newton's second law
Lagrange's equations

III. DERIVATIONS
Heuristic derivation of action principles.
Derivation of Lagrange's equations from action principle.

IV. BRIEF HISTORY
Tracing only those threads that lead to the modern understanding of action.
Fermat's principle: one original variational formulation still in use
General variational problems: Bernoulli brothers, Isaac Newton, etc
Euler's formulation of the general problem in mechanics
Lagrange swamps the field.
Hamilton's action
Euler analysis reborn.
Role of computers in finding worldline with stationary value of the action.

V. GENERALIZED COORDINATES, CONSTRAINED MOTION, AND NONCONSERVATIVE SYSTEMS

VI. MULTIPARTICLE SYSTEMS AND RIGID BODIES

VII. RECENT DEVELOPMENTS

Gray, Karl, and Novikov expand the two action principles discussed here (Maupertuis-Euler abbreviated action and Hamilton action) to four, adding the principle of stationary abbreviated action at constant *mean* energy and the principle of stationary mean energy at constant abbreviated action. The latter is the classical limit of Schroedinger's variational principle of wave mechanics. "The four variational principles . . . form a complete set of principles of mechanics invariant under reciprocity and Legendre transformations." Further study will determine the degree of Resource Letter emphasis to on their innovations and other new results. Reference: C. G. Gray, G. Karl, and V. A. Novikov, "Progress in classical and quantum variational principles," *Reports on Progress in Physics,* 67 (2004), pp 159-208.

VIII. ACTION PRINCIPLES IN OTHER FIELDS

A few words on each; mostly commentary on bibliography:
Foundation of action principle in Feynman's formulation of quantum mechanics.
Principle of maximal aging in special relativity and for time-independent metrics in GR.
Electromagnetism and circuit theory.
Classical atomic theory (Max Born).
Old quantum theory (Sommerfeld).
Current research applications.

VIII. ANNOTATED BIBLIOGRAPHY

A. General treatments and formal histories
(Lanczos, Yourgrau & Mandelstam, Goldstine history etc.)

B. Standard Textbooks.

C. Relevant journal articles of last few decades.

D. Original documents

E. Applications of action principles to other fields

TECHNICAL NOTES

Book references include ISBNs where available.
Past articles in *American Journal of Physics* from increasingly remote dates are available online.
All of Hamilton's articles are available online.
Other online resources will be sought and presented.