SpaceTime Physics: Introduction to Special Relativity, Second Edition

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THE UNITY OF SPACETIME

Relativity describes Nature from quark to cosmos. Relativity empowers its user to ponder deeply, to analyze widely, to predict accurately. It is a theory of fantastic innocence, simplicity, and power.

Yet "relativity theory" is a misleading term, a term Albert Einstein avoided for years. True, he recognized and revealed to the world that the time between two events is typically different as recorded by Earth observer or spaceship commander. Time between events is relative. Relative too is the distance between events. Yet behind these differences Einstein discerned unity: concepts and quantities on which everyone in the universe agrees. What concepts and quantities?

Events. An explosion is an explosion. A birth is a birth. Whether it is the birth of a star or your own birth, everyone agrees that it happens.

Wristwatch time. Carry a wristwatch directly from one event to a second event, so that both take place at the wristwatch. Or lay a rod between two events that occur at the same time. Everyone correctly predicts the wristwatch reading and this rod length.

The path connecting events. Were you, there, at the first event? Yes. And at the second? Yes. And the last? Yes. Does everyone in the universe agree that you were present at every event in this string? Yes. Does everyone agree on the advance of your wristwatch time from event to event along this entire string of events? Yes!

Conservation laws. Everyone agrees that momentum is conserved in a collision of particles. It is also conserved when particles are created, transformed, or annihilated in that collision. Energy, too, is conserved in the same collision, everyone agrees. •

Agreements of these four kinds bear witness to a powerful and simple unity, the unity of space and time: spacetime! Special relativity explores the unity of spacetime. General relativity recognizes that spacetime is not just a passive stage on which events occur; spacetime is an actor that takes part in physical events. All of relativity comes in a single simple sentence: Spacetime grips mass, telling it how to move, and mass grips spacetime, telling it how to curve.
Both males and females make competent observers. We ordinarily treat the laboratory observer as male and the rocket observer as female. Beyond this, to avoid alternating "his" and "her" in a single chapter, we use female pronouns for an otherwise undesignated observer in odd-numbered chapters and male pronouns in even-numbered chapters.

Epigram, facing page: Einstein remark to his assistant Ernst Straus, quoted in Mainsprings of Scientific Discovery by Gerald Holton in The Nature of Scientific Discovery, Owen Gingerich, Editor (Smithsonian Institution Press, Washington, 1975).
What I'm really interested in is whether God could have made the world in a different way; that is, whether the necessity of logical simplicity leaves any freedom at all.

— Albert Einstein
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SELECTED PHYSICAL CONSTANTS

Speed of light in a vacuum
\[ \begin{align*}
\text{Speed of light in a vacuum} & \quad c = 2.99792458 \times \left\{ \begin{array}{l}
10^8 \text{ meters/second} \\
10^{10} \text{ centimeters/second}
\end{array} \right. \\
\text{1 meter of distance/meter of light-travel time} & \quad 1 \text{ centimeter of distance/centimeter of light-travel time}
\end{align*} \]

Gravitational constant
\[ G = 6.673 \times \left\{ \begin{array}{l}
10^{-11} \text{ meter}^3/(\text{kilogram} \cdot \text{second}^2) \\
10^{-8} \text{ centimeter}^3/(\text{gram} \cdot \text{second}^2)
\end{array} \right. \]

Planck constant
\[ h = 6.6261 \times \left\{ \begin{array}{l}
10^{-34} \text{ kilogram} \cdot \text{meter}^2/\text{second} \\
10^{-15} \text{ gram} \cdot \text{centimeter}^2/\text{second}
\end{array} \right. \]

Boltzmann constant
\[ k = 1.38066 \times \left\{ \begin{array}{l}
10^{-23} \text{ joule/degree Kelvin} \\
10^{-16} \text{ erg/degree Kelvin}
\end{array} \right. \]

Elementary charge
\[ e = \left\{ \begin{array}{l}
1.60218 \times 10^{-19} \text{ coulombs} \\
4.80321 \times 10^{-10} \text{ esu or (gram centimeter}^2/\text{second}^3)^{1/2}
\end{array} \right. \]

Electron mass
\[ m_e = 9.1094 \times \left\{ \begin{array}{l}
10^{-11} \text{ kilogram} \\
10^{-21} \text{ gram}
\end{array} \right. \]

Electron rest energy
\[ m_e c^2 = 8.1871 \times \left\{ \begin{array}{l}
10^{-11} \text{ joules} \\
10^{-7} \text{ ergs}
\end{array} \right. \]
\[ = 0.510999 \text{ MeV} \]

Proton mass
\[ m_p = 1.67262 \times \left\{ \begin{array}{l}
10^{-27} \text{ kilogram} \\
10^{-21} \text{ gram}
\end{array} \right. \]

Proton rest energy
\[ m_p c^2 = 1.503279 \times \left\{ \begin{array}{l}
10^{-14} \text{ joules} \\
10^{-3} \text{ ergs}
\end{array} \right. \]
\[ = 938.272 \text{ MeV} \]

Mass of Earth
\[ M_{\oplus} = 5.9742 \times \left\{ \begin{array}{l}
10^{24} \text{ kilograms} \\
10^{32} \text{ grams}
\end{array} \right. \]

Radius of a sphere having the same volume as Earth
\[ R_{\oplus} = 6.3710 \times \left\{ \begin{array}{l}
10^{8} \text{ meters} \\
10^{10} \text{ centimeters}
\end{array} \right. \]

Mean distance of Earth from Sun = "astronomical unit"
\[ AU = 1.495978 \times \left\{ \begin{array}{l}
10^{11} \text{ meters} \\
10^{13} \text{ centimeters}
\end{array} \right. \]

Mean speed of Earth in its orbit about Sun
\[ v = 29.78 \text{ kilometers/second} \]

Mean distance of Moon from Earth
\[ 3.844 \times \left\{ \begin{array}{l}
10^{8} \text{ meters} \\
10^{10} \text{ centimeters}
\end{array} \right. \]

Mass of Sun
\[ M_\odot = 1.989 \times \left\{ \begin{array}{l}
10^{30} \text{ kilograms} \\
10^{33} \text{ grams}
\end{array} \right. \]

Mean radius of Sun
\[ R_\odot = 6.9599 \times \left\{ \begin{array}{l}
10^{8} \text{ meters} \\
10^{10} \text{ centimeters}
\end{array} \right. \]

Conversion Factors

1 second = 2.99792458 × \left\{ \begin{array}{l}
10^8 \text{ meters} \\
10^{10} \text{ centimeters}
\end{array} \right\} of light-travel time

1 meter of light-travel time = 3.335641 × 10^{-9} \text{ second}

1 centimeter of light-travel time = 3.335641 × 10^{-11} \text{ second}

1 year = 3.156 × 10^7 \text{ seconds} = 9.460 × \left\{ \begin{array}{l}
10^{15} \text{ meters} \\
10^{17} \text{ centimeters}
\end{array} \right\} of light-travel time

1 kilometer = 0.6214 mile

1 electron-volt = 1.602 × 10^{-19} \text{ joule} = 1.602 × 10^{-12} \text{ erg}
THE AUTHORS

JOHN ARCHIBALD WHEELER (Ph.D., Johns Hopkins University) is one of the world's foremost relativists. He is Joseph Henry Professor Emeritus at Princeton University and, until his retirement in 1986, Blumberg Professor of Physics and Director, Center for Theoretical Physics, at the University of Texas at Austin. A past president of the American Physical Society, he is a recipient of the Enrico Fermi Award (1968), the National Medal of Science (1971), and the Niels Bohr International Gold Medal (1982).

Since the appearance of the First Edition of Spacetime Physics, John Wheeler has published a graduate text in general relativity, GRAVITATION, with Kip S. Thorne and Charles W. Misner (W. H. Freeman, 1970), and a popular treatment of gravity, A Journey into Gravity and Spacetime (Scientific American Library, 1990; distributed by W. H. Freeman).

EDWIN E. TAYLOR (Ph.D., Harvard University) taught physics for 26 years at the Massachusetts Institute of Technology. He is currently Research Professor in the Department of Physics at Boston University. He is the author of a textbook on introductory mechanics and An Introduction to Quantum Physics with A. P. French (W. W. Norton, 1978). He has served as Editor of the American Journal of Physics.

With MIT undergraduates, Edwin Taylor produced interactive computer programs to help students visualize and solve problems in special relativity. These won the 1988 EDUCOM/NCRIPTAL Higher Education Software Awards for Best Physics Software and Best Tool Software.

THE BOOK

Collaboration on the First Edition of Spacetime Physics began in the mid-1960s when Edwin Taylor took a junior faculty sabbatical at Princeton University where John Wheeler was a professor. The resulting text emphasized the unity of space-time and those quantities (such as proper time, proper distance, mass) that are invariant, the same for all observers, rather than those quantities (such as space and time separations) that are relative, different for different observers. The text has become a standard for modern physics and relativity courses, as well as introductory physics.

The Second Edition of Spacetime Physics embodies what the authors have learned during an additional quarter century of teaching and research. They have updated the text to reflect the immense strides in physics during the same period and modernized and increased the number of exercises, for which the First Edition was famous. Enrichment boxes provide expanded coverage of intriguing topics. Sample problems encourage students to exercise their newfound power. An enlarged final chapter on general relativity includes new material on gravity waves, black holes, and cosmology.

The Second Edition of Spacetime Physics provides a new generation of students with a deep and simple overview of the principles of relativity.