CHAPTER 6

REGIONS OF SPACETIME

6.1 LIGHT SPEED: LIMIT ON CAUSALITY

no signal reaches us faster than light

Nine-year-old Meredith waves her toy magician's wand and shouts, "Sun is exploding right now!" Is she right? We have no way on Earth of knowing—at least not for a while. Sun lies 150,000 million meters from Earth. Therefore it will take 150,000 million meters of light-travel time for the first light flash from the explosion to reach us. This equals 500 seconds—8 minutes and 20 seconds. We will just have to wait and see if Meredith is correct . . .

Signal Sun with super speed?

When 8 minutes and 20 seconds pass, we have evidence that Meredith was mistaken: Looking through our special dark glasses, we see no exploding Sun.

But Meredith's wand has started us thinking. What in the laws of nature prohibits the wave of her wand from being the signal for Sun to explode at that same instant? Or—more reasonably, given the awesome event—what prevents Meredith from having instantaneous warning, so that she raises her wand simultaneously with Sun's explosion in order to give us (in light of later developments) a false impression of her power?

Both questions have the same answer: "The speed of light." Whatever her powers, Meredith cannot affect Sun in less than 500 seconds; neither can a warning signal reach us from Sun in less time than that. All during that intervening 500 seconds we would see the accustomed round shape of Sun, apparently healthy as ever.

No, just speed of light

More generally, one event cannot cause another when their spatial separation is greater than the distance light can travel in the time between these events. Light speed sets a limit on causality. No known physical process can overcome this limit: not gravity, not some other field, not a zooming particle of any kind. "Spacetime interval" quantifies this limit on causality. Interval between far-away events—unlike distance between far-away points—can be zero. In this and other ways the spacetime geometry of the real world differs fundamentally from the space geometry of Euclid's 2300-year-old textbook.

6.2 RELATION BETWEEN EVENTS: TIMELIKE, SPACELIKE, OR LIGHTLIKE

minus sign yields three possible relations between pairs of events

Using Euclidean geometry, a surveyor reckons the distance between two steel stakes from the sum of the squares of the northward and eastward separations of these stakes:

uared distance: Positive or zero

$$(\text{distance})^2 = (\text{northward separation})^2 + (\text{eastward separation})^2$$

In consequence, in Euclidean geometry a distance—or its square—always has a positive value or zero.

In contrast, the spacetime interval between events in Lorentz geometry arises from the *difference* of squares of time and space separations:

Squared interval: Positive, zero, or negative

$$(\text{interval})^2 = (\text{separation in time})^2 - (\text{separation in space})^2$$

In consequence of the minus sign, this equation yields a number that may be positive, negative, or zero, depending on whether the time or the space separation predominates. Moreover, whichever of these three descriptions characterizes the interval in one free-float frame also characterizes the interval in any other free-float frame. Why? Because the spacetime interval between two events has the same value in all overlapping free-float frames. In the threefold possibilities for an interval, nature reveals the causal relation between events.

An interval between two events earns the name **timelike** or **spacelike** or **lightlike** depending on whether the time part predominates, the space part predominates, or the time and space parts are equal, respectively, as shown in Table 6-1. For convenience, the minus sign is placed so that the resulting squared interval is greater than or equal to zero.

Timelike interval: Time part dominates

Timelike Interval: We picture the sequence of sparks emitted by a moving sparkplug. Points representing these sparks on the spacetime map trace out the worldline of the particle (Chapter 5). No material particle has ever been measured to travel faster than light. Every material particle always travels *less* than one meter of distance in one meter of light-travel time. The sparks emitted by the particle have a greater time separation than their separation in space. In other words, the worldline of a particle consists of events that have a timelike relation with one another and with the initial event. We say that a material particle follows a **timelike worldline.**

The interval τ between two timelike events reveals itself to the observer in any free-float frame:

$$(\text{timelike interval})^2 = \tau^2 = (\text{time separation})^2 - (\text{space separation})^2 \qquad (6\text{-}1)$$

TABLE 6-1

CLASSIFICATION OF THE RELATION BETWEEN TWO EVENTS

Description	Squared interval is named and reckoned
Time part of interval dominates space part	$(\text{timelike interval})^2 = \tau^2 = (\text{time})^2 - (\text{distance})^2$
Space part of interval dominates time part	$(\text{spacelike interval})^2 = s^2 = (\text{distance})^2 - (\text{time})^2$
Time part of interval equals space part	$(\text{lightlike interval})^2 = 0 = (\text{time})^2 - (\text{distance})^2$

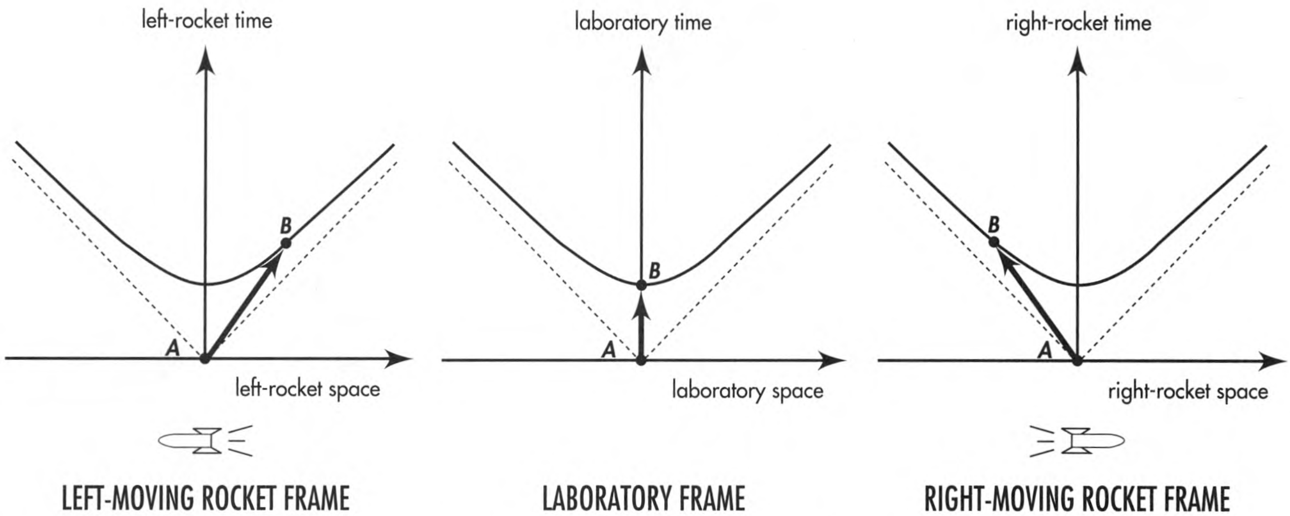


FIGURE 6-1. ***Events A and B form a timelike pair (with event A arbitrarily chosen as reference event), here recorded in the spacetime maps of three free-float frames.*** *Point B lies on a hyperbola opening along the time axis in each frame. The* shortest *time between events A and B is recorded in the laboratory frame, the frame in which the two events occur at the same place.*

Same two sparks registered in different frames? Different records for the separation in time between those sparks. Different records for the separation in space. *Same* figure for the *timelike interval* between them!

Nobody can keep us from tracing out on one and the same diagram (Figure 6-1) the very different records for the separation AB that observers get in different free-float frames. One frame? One point on the diagram. Another frame? Another point on the diagram. And so on. These many records for the same pair of events AB trace out a hyperbola. This hyperbola opens out in the time direction.

Timelike interval: Invariant hyperbola opens along time axis

The two sparks, A and B—definite locations though they occupy in spacetime—nevertheless register in different frames of reference as having different separations in reference-frame time. Among the many conceivable frames, which one records this separation in time as smallest? Answer: The frame in which spark B occurs at the same *place* as spark A. In other words, the frame that happens to move along in sync with the sparkplug, even if only briefly. In that frame the clock records a separation in time between A and B identical with the timelike interval AB.

As seen in the left-moving rocket frame in Figure 6-1, spark B lies to the right of spark A. In contrast, spark B occurs to the left of spark A in the right-moving rocket. The position of B relative to A depends on the reference frame from which it is measured. For a pair of events separated by a timelike interval, labels "right" and "left" have no invariant meaning: they are frame-dependent.

Spacelike interval: Space part dominates

Spacelike Interval: The interval between two events A and D is spacelike when the space part predominates over the time part. Such was the case for a possible explosion of Sun (event A) and Meredith's wand waving (event D), simultaneous with A as recorded in the Earth frame (Section 6.1). Events A and D, if they occurred, would be separated in the Earth–Sun frame by a distance of 150,000 million meters and separated by a time of zero meters. Clearly the space part predominates over the time part! Whenever the space part predominates, we call the relation between the two events *spacelike*.

The interval s (sometimes called by the Greek letter sigma, σ) between two spacelike events reveals itself to the observer in any free-float frame:

$$(\text{spacelike interval})^2 = s^2 = (\text{space separation})^2 - (\text{time separation})^2 \qquad (6\text{-}2)$$

Events A and D registered in different frames? Then different records for the separation in time between those events. Also different records for the separation in space. *Same* numerical value for the *spacelike interval* between them!

Spacelike interval: Invariant hyperbola opens along space axis

We plot on another spacetime diagram (Figure 6-2) all of the very different records for the separation AD that observers get in different free-float frames. One frame? One point on the diagram. Another frame? Another point on the diagram. And so on. These many records for the same pair of events AD trace out a hyperbola. This hyperbola opens out in the space direction.

The two events, A and D—definite locations though they occupy in spacetime—nevertheless register in different frames of reference as having different separations in reference-frame space. Among the many conceivable frames, which one records this separation in space as smallest? Answer: The frame in which spark D occurs at the same *time* as spark A. In that frame a long stick records a separation in space between A and D identical with the spacelike interval, AD. This is called the **proper distance** between the two spacelike events.

In the Earth–laboratory frame in Figure 6-2, Meredith waves her wand (event D) at the same time as Sun explodes (event A). In the right-moving rocket frame Sun explodes *after* Meredith waves her wand. In the left-moving rocket frame Sun explodes *before* the wand wave. For a pair of events separated by a spacelike interval, labels "before" and "after" have no invariant meaning: they are frame-dependent. To allow the wand to control Sun would be to scramble cause and effect!

No particle—not even a flash of light—can move between two events connected by a spacelike interval. To do so would require it to cover a distance greater than the time available to cover this distance (space separation greater than time separation). In brief, it would have to travel faster than light. This is alternative evidence that two events separated by a spacelike interval cannot be causally connected: one of them cannot "get at" the other one by any possible signal.

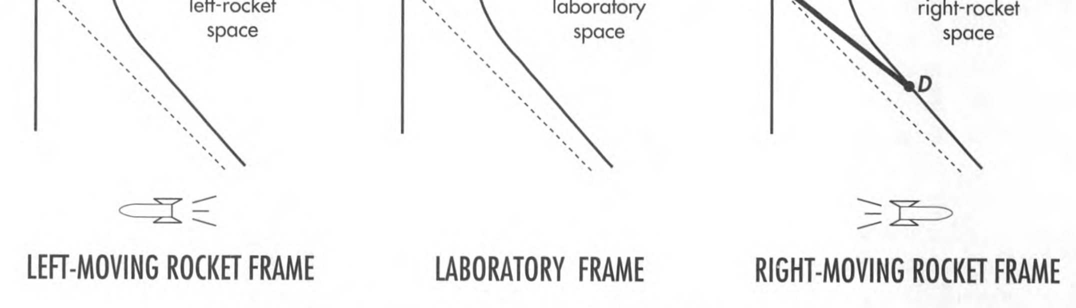


FIGURE 6-2. *The spacelike pair of events* A *and* D *(with event* A *arbitrarily chosen as reference event) as recorded in the spacetime maps of three free-float frames.* *Point* D *lies on a hyperbola opening along the space axis in every rocket and laboratory frame. The shortest* distance *between these events is recorded in the laboratory frame, the frame in which the two events occur at the same time. A heavy line represents the spacetime separation* AD. *No particle can travel along this line; the speed would be greater than light speed—and would be infinitely great as measured in the laboratory frame, since the particle would have to cover the distance from* A *to* D *in zero time!*

SAMPLE PROBLEM 6-1

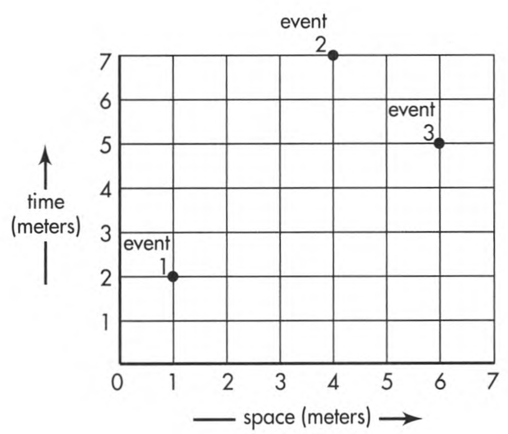
RELATIONS BETWEEN EVENTS

Events 1, 2, and 3 all have laboratory locations $y = z = 0$. Their x and t measurements are plotted on the laboratory spacetime map.

a. Classify the interval between events 1 and 2: timelike, spacelike, or lightlike.

b. Classify the interval between events 1 and 3.

c. Classify the interval between events 2 and 3.



SOLUTION

a. For event 1, $t = 2$ meters and $x = 1$ meter. For event 2, $t = 7$ meters and $x = 4$ meters. The squared interval between them: $(\text{interval})^2 = (7-2)^2 - (4-1)^2 = 5^2 - 3^2 = 25 - 9 = 16\ (\text{meters})^2$. The time part is greater than the space part, so the interval between these two events is *timelike:* $\tau = 4$ meters.

b. For event 1, $t = 2$ meters and $x = 1$ meter. For event 3, $t = 5$ meters and $x = 6$ meters. The squared interval between them: $(\text{interval})^2 = (5-2)^2 - (1-6)^2 = 3^2 - 5^2 = 9 - 25 = -16\ (\text{meters})^2$. The space part is greater than the time part, so the interval is *spacelike:* $s = 4$ meters. (For spacelike intervals, we subtract the squared time part from the squared space part before taking the square root.)

c. For event 2, $t = 7$ meters and $x = 4$ meters. For event 3, $t = 5$ meters and $x = 6$ meters. The squared interval between them: $(\text{interval})^2 = (7-5)^2 - (4-6)^2 = 2^2 - 2^2 = 4 - 4 = 0\ (\text{meters})^2$. The time part equals the space part, so the interval is *lightlike*: it is a null interval.

Lightlike Interval (Null Interval): Two events stand in a lightlike relation when the interval between them is zero:

$$(\text{time separation})^2 - (\text{space separation})^2 = 0$$

Lightlike interval: Time separation equa space separation

or

$$\text{magnitude of (separation in time)} = (\text{distance in space}) \quad \text{[for lightlike interval]} \quad (6\text{-}3)$$

Lightlike interval: Plotted along ± 45 degree lines

An interval that is lightlike? A separation in time between two events, A and G, identical to the distance in space between them? What does this condition mean? This: A pulse of light can fly directly from event A and arrive with perfect timing at event G. How come? Distance in meters between the two locations measures the meters of time *required* for light to fly from one place to the other. Separation in time between the two events represents the time *available* for the trip. Time available equals time needed? Guarantee that the pulse from A arrives in coincidence with event G! More generally, whenever the influence of one event, spreading out at the speed of light, can directly affect a second event, then the interval between those two events rates as lightlike, zero, null.

Only light ("photons"), neutrinos, and gravitons can move directly between two events connected by a lightlike interval. Only by means of one of these light-speed particles can the one event in a lightlike pair cause the other.

The spherical out-going pulse of light from an event, A, may trigger two widely separated events, E and G (Figure 6-3). Does this common genesis imply that E and G occur at the same time? Yes and no! Yes, there's always a free-float reference frame in which the two daughter events appear as simultaneous. That frame — for no good reason — we call the laboratory frame in Figure 6-3. In other frames of reference — for example, the left-moving rocket frame in Figure 6-3 — the clocks show that E occurs before G. There are still other frames — the right-moving rocket frame is one — in which the clocks register E and G in the opposite order of time. But no frame shows either E or G in the past of A.

Hold it! Aren't spacelike separations impossible? I understand timelike and lightlike separations between two events, because a particle — or at least a light flash — can travel between them. Not even a light flash, however, can travel from one event to a second event separated from the first by an interval that is spacelike. The first event cannot possibly cause the second event in the spacelike case. Therefore a spacelike interval cannot arise in nature. So why talk about it?

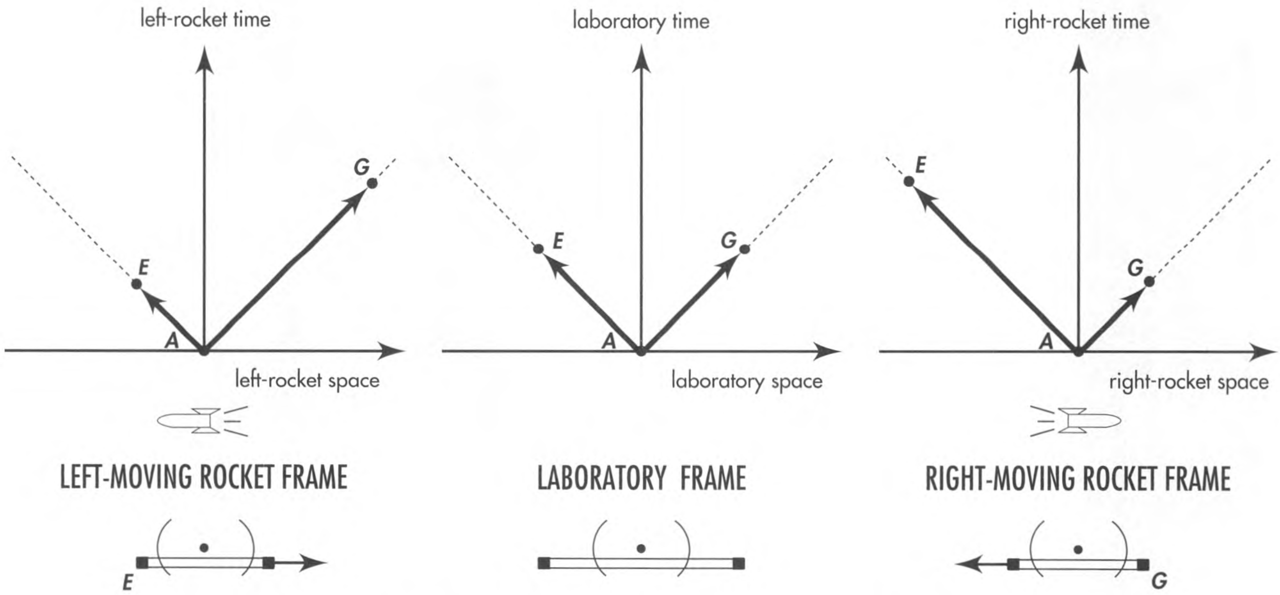


FIGURE 6-3. ***Two lightlike pairs of events* AE *and* AG *(with event* A *arbitrarily chosen as reference event) as recorded in spacetime maps of three free-float frames.*** *A flash originates at* A *and spreads outward from the center of a rod at rest in the laboratory frame. Events* E *and* G *are receptions of this flash at the two ends of the rod as recorded by different observers. In the laboratory frame, reception events* E *and* G *occur at the same time. In the right-moving rocket frame, the rod moves to the left, so event* G *occurs sooner than event* E. *In the left-moving rocket frame, the rod moves to the right, so event* E *occurs sooner than event* G.

Oops! A spacelike interval between two events certainly can and does arise in nature.

Signals from the supernova labeled 1987A reported that event to us in 1987, which was 150,000 years after the explosion occurred. Yet occur it did! No astronomer of Babylonian, Egyptian, or Greek days reported it, nor could they even know of it. Yet it had already happened for them. That event separated itself from each of them by a spacelike interval. Only the advance of time to the year 1987 brought down the interval between that explosion and Earthbound observers from spacelike to lightlike. In that year a light pulse carried the earliest possible report of that explosion to our eyes. And look today? See no explosion at that location in the sky. The light from it has passed us by. Our present relation to that event? Timelike!

6.3 LIGHT CONE: PARTITION IN SPACETIME

invariance of the interval preserves cause and effect

Thus far in dealing with the interval between two events, *A* and *B*, we have considered primarily the situation in which these events lie along a single direction in space — on the reference line where the laboratory and rocket reference clocks are located. In contrast, the surveyors in our imaginary kingdom made use of two space dimensions — northward and eastward. We know, however, that Euclidean space is truly three-dimensional. A surveyor measuring hilly terrain soon appreciates the need for a third dimension: the direction vertically upward! The measure of distance in three dimensions requires a simple extension of the expression for distance in two dimensions: The square of the distance becomes the sum of the squares of *three* mutually perpendicular separations:

Interval generalized to three space dimensions

$$(\text{distance})^2 = (\text{north separation})^2 + (\text{east separation})^2 + (\text{up separation})^2$$

Euclidean space requires three dimensions. In contrast, spacetime, which includes the time dimension, demands four. The expression for the square of a timelike interval now has four terms: a positive term (the square of the time separation) and three negative terms (the squares of the separations in three space dimensions).

$$(\text{interval})^2 = (\text{time separation})^2 - (\text{north separation})^2 - (\text{east separation})^2 - (\text{up separation})^2$$

The three space terms can be represented by the single distance term in the equation above, yielding

$$\begin{aligned}(\text{timelike interval})^2 &= (\text{time separation})^2 - (\text{distance})^2\\ (\text{spacelike interval})^2 &= (\text{distance})^2 - (\text{time separation})^2\\ (\text{lightlike interval})^2 &= 0 = (\text{time separation})^2 - (\text{distance})^2\end{aligned}$$

or, for the lightlike interval,

$$\text{magnitude of (separation in time)} = \text{(distance in space)} \qquad \text{[lightlike interval]} \quad (6\text{-}3)$$

For pairs of events with lightlike separation, the interval equals zero. The zero interval is a unique feature of Lorentz geometry, new and quite different from

SAMPLE PROBLEM 6-2

EXPLETIVE DELETED

At 12:00 noon Greenwich Mean Time (GMT) an astronaut on Moon drops a wrench on his toe and shouts "Damn!" into his helmet microphone (event A), carried by a radio signal toward Earth. At one second after 12:00 noon GMT a short circuit (event D) temporarily disables the receiving amplifier at Mission Control on Earth. Take Earth and Moon to be 3.84×10^8 meters apart in the Earth frame and assume zero relative motion.

a. Does Mission Control on Earth hear the astronaut's expletive?

b. Could the astronaut's strong language have caused the short circuit on Earth?

c. Classify the spacetime separation between events A and D: timelike, spacelike, or lightlike.

d. Find the proper distance or proper time between events A and D.

e. For all possible rocket frames passing between Earth and Moon, find the shortest possible distance between events A and D. In the rocket frame for which this distance is shortest, determine the time between the two events.

SOLUTION

a. In one second, electromagnetic radiation (light and radio waves) travels 3.0×10^8 meters in a vacuum. Therefore the radio signal does not have time to travel the 3.84×10^8 meters between Moon and Earth in the one second available between the events A and D as measured in the Earth frame. So Mission Control does not hear the exclamation.

b. No signal travels faster than light. So the astronaut's strong language cannot have caused the short circuit.

c. The space part of the separation between events (3.84×10^8 meters) dominates the time part (one second $= 3.0 \times 10^8$ meters). Therefore the separation is spacelike.

d. The square of the proper distance s comes from the expression

$$\begin{aligned} s^2 &= (\text{space separation})^2 - (\text{time separation})^2 \\ &= (3.84 \times 10^8 \text{ meters})^2 - (3.00 \times 10^8 \text{ meters})^2 \\ &= (14.75 - 9.00) \times 10^{16} \text{ (meters)}^2 \\ &= 5.75 \times 10^{16} \text{ (meters)}^2 \end{aligned}$$

The proper distance equals the square root of this value: $s = 2.40 \times 10^8$ meters

e. The proper distance equals the shortest distance between two spacelike events as measured in any rocket frame moving between them (Figure 6-2, laboratory map). Hence 2.40×10^8 meters equals the shortest possible distance between events A and D. In the particular rocket frame for which the distance is shortest, the time between the two events has the value zero—events A and D are simultaneous in this frame.

SAMPLE PROBLEM 6-3

SUNSPOT

Bradley grabs his sister's wand and waves it, shouting "Sunspot!" At that very instant his father, Lloyd, who is operating a home solar observatory, sees a spot appear on the face of Sun. Let event E be Bradley waving the wand and event A be eruption of the sunspot at the surface of Sun itself. The Earth–Sun distance equals approximately 1.5×10^{11} meters. Neglect relative motion between Earth and Sun.

a. Is it possible that Bradley's wand waving caused the sunspot to erupt on Sun?

b. Is it possible that the sunspot erupting on Sun caused Bradley to wave his wand?

c. Classify the spacetime separation between events A and E: timelike, spacelike, or lightlike.

d. Find the value of proper distance or proper time between events A and E.

e. For all possible rocket frames passing between Earth and Sun, find the shortest possible distance or the shortest possible time between events A and E.

SOLUTION

a. Light travels 1 meter of distance in 1 meter of time—or 1.5×10^{11} meters of distance in 1.5×10^{11} meters of time. Hence in the Earth-Sun frame, eruption of the sunspot (event A) occurred 1.5×10^{11} meters of time *before* Bradley waved the wand (event E). So Bradley's wand waving could not have caused the eruption on Sun.

b. On the other hand, it is possible that eruption of the sunspot caused Bradley to wave his wand: He raises the wand in the air, looks over his father's shoulder, and waves the wand as the spot appears on the projection screen. (We neglect his reaction time.)

c. Events A and E are connected by one light pulse; their space and time separations both have the value 1.5×10^{11} meters in the Earth frame. Therefore the spacetime separation between them is lightlike.

d. Space and time separations between events A and E are equal. Therefore the interval between them has value zero. Hence proper time between them—equal to proper distance between them—also has value zero.

e. The interval is invariant. Therefore all possible free-float rocket frames passing between Earth and Sun reckon zero interval between events A and E. This means each of them measures space separation between events A and E equal to the time separation between these events. The common value of the space and time separations are not the same for all rocket frames, but they are equal to one another in every individual rocket frame. We are asked to find the shortest possible value for this time.

Think of a rocket just passing Sun as the sunspot erupts, the rocket headed toward Earth at nearly light speed with respect to Earth. Rocket lattice clocks record the light flash from the sunspot moving away from the rocket at standard speed unity. However, these clocks record that Earth lies very close to Sun (Lorentz contraction of distance) and that Earth rushes toward the rocket at nearly light speed. Therefore light does not travel far to get to Earth in this rocket frame; neither does it take much time. For a rocket moving arbitrarily close to light speed, this distance between A and E approaches zero, and so does the time

SAMPLE PROBLEM 6-3

between A and E. Hence the shortest possible distance between A and E—equal to the shortest possible time between A and E—has the value zero. But this constitutes a limiting case, since rocket speed may approach but cannot equal the speed of light in any free-float frame.

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anything in Euclidean geometry. In Euclidean geometry it is never possible for distance AG between two points to be zero unless all three of the separations (northward, eastward, and upward) equal zero. In contrast, interval AG between two events can vanish even when separation in space and separation in time are individually quite large. Equation (6-3) describes the separation between lightlike events, but now separation in space may show up in two or three space dimensions as well as one time dimension. The distance in space is always positive.

Light flash traces out light cone in spacetime diagram

It is interesting to plot on an appropriate map locations of all events, G, G_1, G_2, G_3, . . . , that can be connected with one given event A by a single spreading pulse of light. Every such future event has a distance in space from A identical to its delay in time after A. Only so can it satisfy the requirement (6-3) for a null interval. For it:

$$\text{(future time with respect to } A) = +\text{(distance in space from } A) \quad \text{[lightlike interval]} \quad (6\text{-}4)$$

It is equally interesting to display—and on the same diagram—all the events H, H_1, H_2, H_3, . . . that can send a light pulse to A. Every such event fulfills the condition

$$\text{(past time relative to } A) = -\text{(distance in space from } A) \quad \text{[for lightlike interval]} \quad (6\text{-}5)$$

Both of these equations satisfy the magnitude equation (6-3).

In Figure 6-4 we suppress display of a third space dimension in the interest of simplicity. We limit attention to future events G, G_1, G_2, . . . and past events H, H_1, H_2, . . . that lie on a north–south/east–west plane in space. A flash emitted from event A expands as a circle on this space plane. As it spreads out from event A, this circle of light traces out a cone opening upward in the spacetime map of Figure 6-4. This is called the **future light cone** of event A. The cone opening downward traces the history of an in-coming circular pulse of radiation so perfectly focused that it converges toward event A, collapsing exactly at event A at time zero. This downward-opening cone has the name **past light cone** of event A. All the events G, G_1, G_2, . . . lie on the future light cone of event A, all events H, H_1, H_2, . . . on its past light cone.

Numerous as the events may be that lie on the light cone, typically there are many more that don't! Look, for example, at all the events that occur 7 meters of time later than the zero time of event A. On the spacetime map, these events define a plane 7 meters above the $t = 0$ plane in which event A lies, and parallel to that plane. The light cone intersects this plane in a circle (circle in the present map; a sphere in a full spacetime map with three space dimensions). An event on the plane falls into one or another of three categories, relative to event A, according as it lies inside the circle (as does B in Figure 6-4), on it (as does G), or outside it (as does D).

The light cone is unique to Lorentz geometry. It gives nature a structure beyond any power of Euclidean geometry. The light cone does more than divide events on a single plane into categories. It classifies every event, everywhere in spacetime, into one or another of five distinct categories according to the causal relation that event bears to the chosen event, A:

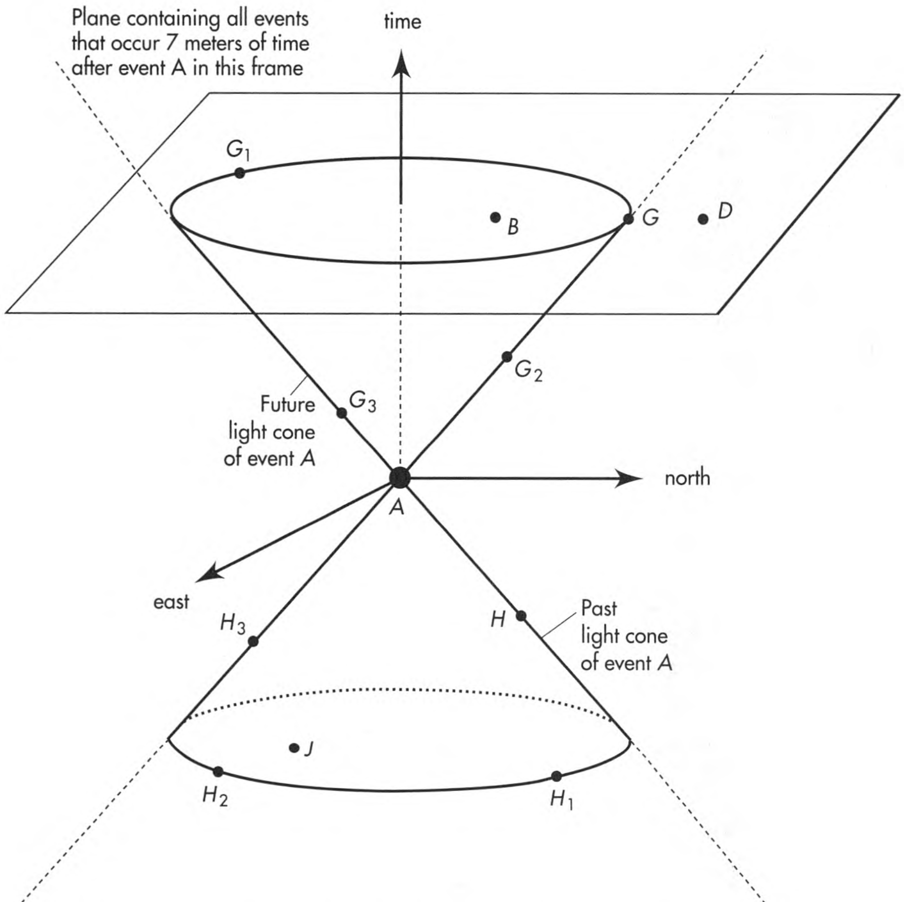


FIGURE 6-4. ***Light cone as partition in spacetime; perspective three-dimensional spacetime map showing eastward, northward, and time locations of events occurring on a flat plane in space.*** *Events* G, G_1, G_2, *and* G_3 *are on the future light cone of event* A; *events* H, H_1, H_2, *and* H_3 *are on its past light cone. See also Figure 6-5.*

Cause and effect preserved by light cone

1. Can a material **particle** emitted at A affect what **is going** to happen at B? If so, B lies ***inside the future light cone*** of A and forms a timelike pair with event A.
2. Can a **light ray** emitted at A affect—with no time to spare—what **is going** to happen at G? If so, G lies ***on the future light cone*** of A and forms a lightlike pair with event A.
3. Can **no effect whatever** produced at A affect what happens at D? If so, D lies outside the future and past light cones of A and forms a spacelike pair with event A. It lies in the ***absolute elsewhere*** of A.
4. Can a material **particle** emitted at J affect what **is happening** at A? If so, J lies ***inside the past light cone*** of A and forms a timelike pair with event A.
5. Can a **light ray** emitted at H affect—with no time to spare—what **is happening** at A? If so, H lies ***on the past light cone*** of A and forms a lightlike pair with event A.

Nature reveals a cause-and-effect structure beyond the vision of Euclidean geometry. The causal relation between an event B and another event A falls into one or the

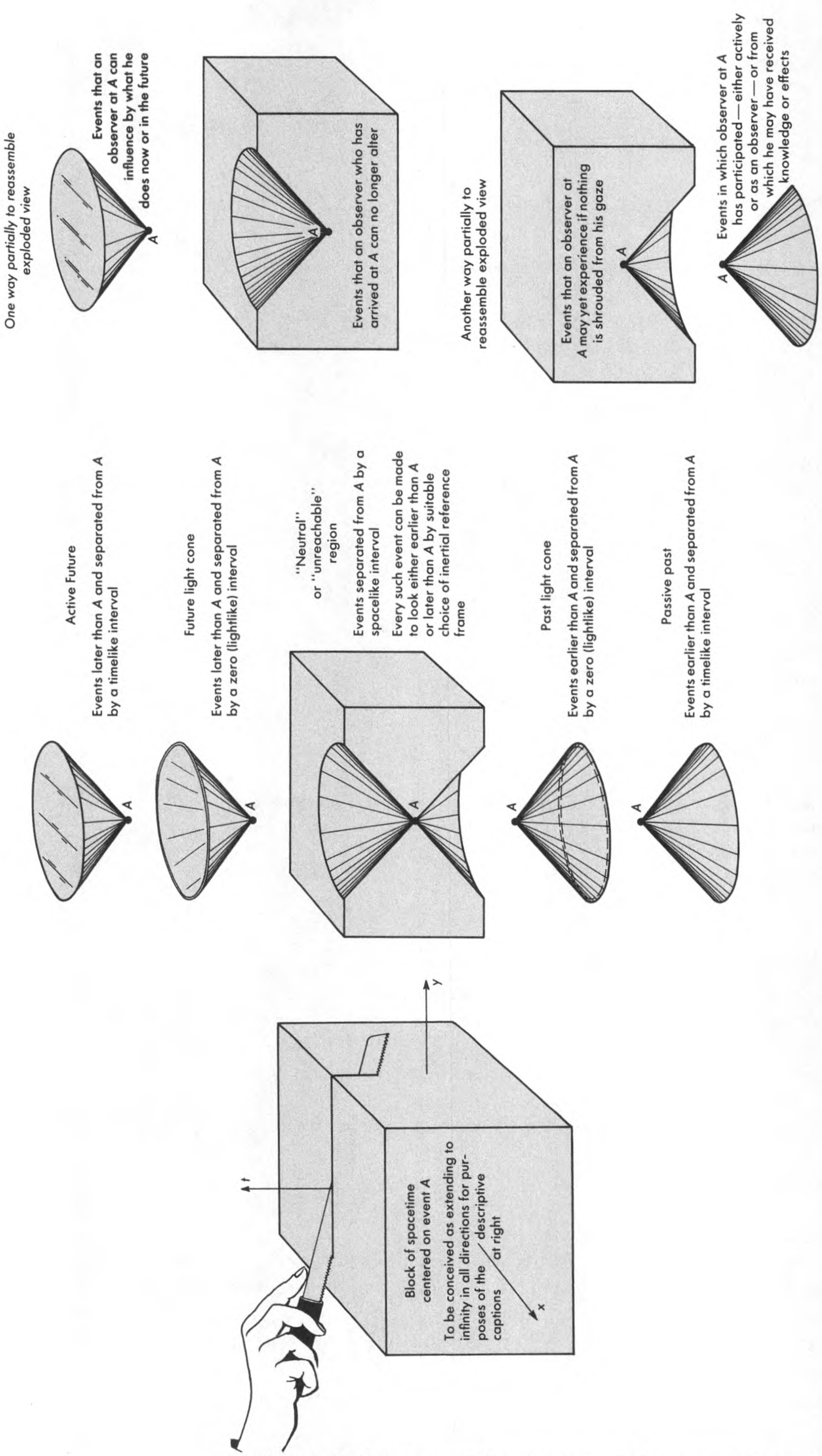


FIGURE 6-5. *Exploded view of the regions into which the events of spacetime fall apart when classified with respect to a selected event* A.

other of five categories picked out by the light cone of A. That light cone and those categories have an existence in spacetime quite apart from any space and time measurements that may be used to describe them. Zero interval between events in one free-float frame means zero interval between the same events in every overlapping free-float frame. The light cone is the light cone is the light cone!

Event A appears at the origin of every spacetime map in this chapter. What's so special about event A?

Nothing whatever is special about event A! On the contrary, we have not captured the full story of the causal structure of spacetime until for *every* event A (A_1, A_2, A_3, . . .) we have classified every *other* event B (B_1, B_2, B_3, . . .) into the appropriate category — timelike! lightlike! spacelike! — with respect to that event.

Figure 6-5 summarizes the relations between a selected event A and all other events of spacetime.

CHAPTER 6 EXERCISES

PRACTICE

6-1 relations between events

This is a continuation of Sample Problem 6-1. Events 1, 2, and 3 all have the laboratory coordinates $y = z = 0$. Their x- and t-coordinates are plotted on the laboratory spacetime diagram.

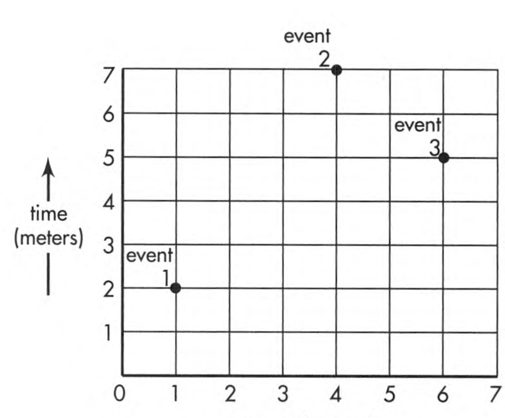
a Answer the following questions three times: once for the timelike pair of events 1 and 2, once for the spacelike pair of events 1 and 3, and once for the lightlike pair of events 2 and 3.

(1) What is the proper time (or proper distance) between the two events?

(2) Is it possible that one of the events caused the other event?

(3) Is it possible to find a rocket frame in which the spatial order of the two events is reversed? That is, is it possible to find a rocket frame in which the event that occurs to the right of the other event in the laboratory frame will occur to the left of the other event in the rocket frame?

(4) Is it possible to find a rocket frame in which the temporal order of the two events is reversed? That is, is it possible to find a rocket frame in which the event that occurs before the other event in the laboratory frame occurs after the other event in the rocket frame?



EXERCISE 6-1. *Laboratory spacetime map.*

b For the timelike pair of events, find the speed and direction of a rocket frame with respect to which the two events occurred at the same place. For the spacelike pair of events, find the speed and direction of a rocket frame with respect to which the two events occurred at the same time.

6-2 timelike, lightlike, or spacelike?

The first table lists the space and time coordinates of three events plus the reference event (event 0) as observed in the laboratory frame.

EXERCISE 6-2

LABORATORY COORDINATES OF THREE EVENTS

	t (years)	x (years)	y (years)
Event 0	0	0	0
Event 1	3	4	0
Event 2	6	5	0
Event 3	8	8	3

a Copy the second table. In the top half of each box in the second table, write the nature of the interval — timelike, lightlike, or spacelike — between the two corresponding events.

b In the bottom half of each box in the second table, write "yes" if it is possible that one of the events caused the other and "no" if it is not possible.

c Find the speed (with respect to the laboratory frame) of a rocket frame in which event 1 and event 2 in the first table occur at the same place.

d Find the speed (with respect to the laboratory frame) of a rocket frame moving along the x-axis in which event 2 and event 3 in the first table occur at the same time.

6-3 proper time and proper distance

Note: This exercise uses the Lorentz transformation equations.

a Two events P and Q have a spacelike separation. Show in general that a rocket frame can be found in which the two events occur at the same time. Also show that in this rocket frame the distance between the two events is equal to the proper distance between them. (One method: assume that such a rocket frame exists and then use the Lorentz transformation equations to show that the relative velocity of this rocket frame is less than the speed of light, thus justifying the assumption made.)

b Two events P and R have a timelike separation. Show in general that a rocket frame can be found in which the two events occur at the same place. Also show that in this rocket frame the time between the two events is equal to the proper time between them.

PROBLEMS

6-4 autobiography of a photon

A photon emitted by a star on one side of our galaxy is absorbed near a star on the other side of our galaxy,

EXERCISE 6-2

INTERVAL BETWEEN EVENTS: TIMELIKE, LIGHTLIKE, OR SPACELIKE?

	Event 1	Event 2	Event 3
Event 0			
Event 1			
Event 2			

100,000 light-years away from its point of origin as measured in the frame of the galaxy. How does the photon experience its own birth and death? That is to say, what are the space and time separations between the birth and death of the photon in the frame of the photon?

Discussion: We cannot answer this question, because we cannot move along with the photon. No matter how fast the unpowered rocket in which we ride, we still measure light to move past us with the speed of light! Still, we can try to answer the question as a limiting case in the galaxy frame. Think of extremely energetic PROTONS traveling the same path. As protons of greater and greater energy are emitted by the first star and are absorbed near the second star at the other side of the galaxy, what happens to the distance between these two events in the frame of the proton? What happens to the time between these events in the frame of the proton? Come in this way to a limiting case in which the PROTON is moving arbitrarily close to the speed of light in the galaxy frame. In this limit, what would you expect the distance and time to be between birth and death in the frame of a PHOTON traveling the same path in space?

a You are the photon. Using the above argument, write the first few sentences of your autobiography.

At the end of the trip, near a star at the fringe of our galaxy, a galaxy-spanning photon travels 10 kilometers vertically through the atmosphere of a planet before it enters a telescope and is absorbed in the eye of an astronomer.

The average **index of refraction** of the atmosphere of this planet is $n = 1.00030$. The speed of the photon in such an atmosphere is $v = v_{\text{conv}}/c = 1/n$. (The speed of light *in a vacuum* is unity.)

b What is the proper time for this last leg of the trip—the time in the rest frame of the "slowed-down" photon? How far apart is the top of the atmosphere and the astronomer's eye in the frame of the photon?

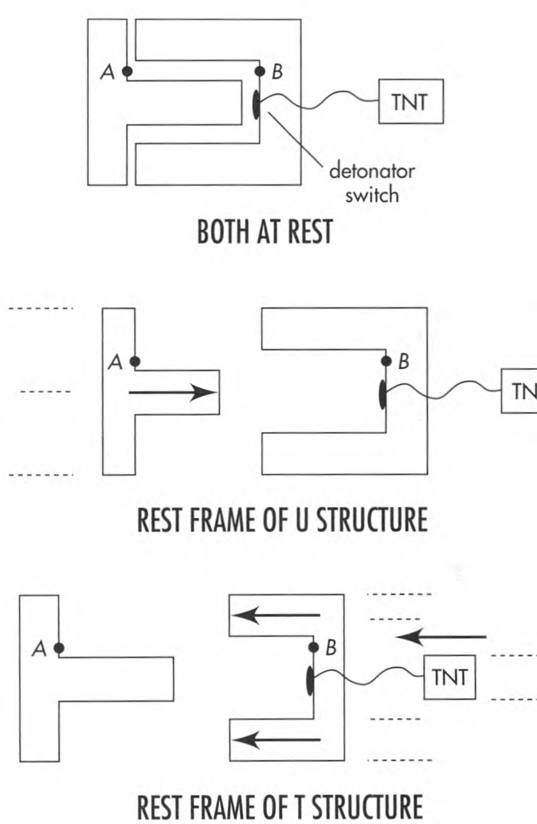
c Complete your photon autobiography with an additional couple of sentences.

Discussion: Relativity is a classical theory—that is, a nonquantum theory—in which photons are postulated to move at light speed in a vacuum and at a speed $v = 1/n$ in air, where n is the index of refraction. **Quantum electrodynamics** (QED), the quantum theory of interactions between light and matter, tells us that it is incorrect to talk of a single photon moving through air. Rather, one thinks of an initial photon being absorbed by an atom in the air and a second photon emitted, the second photon then absorbed by another atom, which emits a third photon, and so forth. The classical relativistic analysis is not correct when viewed from the quantum perspective. For more on quantum electrodynamics, read Richard P. Feynman, *QED: The Strange Theory of Light and Matter* (Princeton, Princeton University Press, 1985).

6-5 the detonator paradox

A U-shaped structure made of the strongest steel contains a detonator switch connected by wire to one metric ton (1000 kilograms) of the explosive TNT, as shown in the figure. A T-shaped structure made of the same strong steel fits inside the U, with the long arm of the T not quite long enough to reach the detonator switch when both structures are at rest in the laboratory.

Now the T structure is removed far to the left and accelerated to high speed. It is Lorentz-contracted along its direction of motion. As a result, its long arm is not long enough to reach the detonator switch when the two collide. Therefore there will be no explosion.



EXERCISE 6-5. ***Both at rest:*** *The leg of the T almost reaches the detonator switch when both the T and the U are at rest. Points* A *and* B *are used in part **b** of the exercise.* ***Rest frame of U structure:*** *The leg of the moving T is Lorentz contracted in the rest frame of the U. Does this mean that the explosion will not take place?* ***Rest frame of T structure:*** *The legs of the moving U are Lorentz-contracted in the rest frame of the T. Does this mean explosion will take place?*

However, look at the same situation in the rest frame of the T structure. In this frame the arm of the T has its rest length, while the two arms of the U structure are Lorentz-contracted. Therefore the arm of the T will certainly strike the detonator switch and there will be a terrible explosion.

a Make a decisive prediction: Will there be an explosion or not? Your life depends on it!

b The wire from the detonator switch to the TNT is restrung through point B on the U structure when both structures are at rest, and a laser is installed at point A on the T structure. Later, when the two structures collide at A, the laser fires a pulse at B that cuts the detonator wire. Does this new apparatus change your prediction about detonation of the TNT?

Acknowledgment: A paper describing this paradox crossed the desk of one of the authors, but the paper and the name of its author have been lost. The laser inhibitor device was devised by Gordon Roesler.

6-6 how fast can you walk?

Webster's Eighth says that to "walk" means to "go on foot without lifting one foot clear of the ground before the other touches the ground." In other words, at least one foot must be on the ground at all times. Use this definition to discover the maximum speed of walking imposed by relativity.

We assume advanced technology here! A walking robot moves its free foot forward at nearly the speed of light. Then one might argue (ambiguously) as follows: While the free foot is moving forward, the planted foot is on the ground, ready to be picked up *when* [look out!] the free foot comes down in front. Half the time each foot is in motion at nearly light speed and half the time it is at rest. Therefore the average speed of each foot, equal to the maximum possible speed of the walking robot, is half the speed of light.

Why is this argument ambiguous? Because of the relativity of simultaneity. The word *when* applied to separated events should always unfurl a red flag. The event "front foot down" (label FrontDown) and the event "rear foot up" (label RearUp) occur at different places along the line of motion. Observers in relative motion will disagree about whether or not events FrontDown and RearUp occur at the same time. Therefore they will disagree about whether or not the robot has one foot on the ground at all times in order to satisfy the dictionary definition of walking.

How to remove the ambiguity in the definition of walking? One way is to make the conventional definition frame-independent: One foot must be on the ground at all times *as observed in every free-float frame of reference*. What limits does this place on the two events FrontDown and RearUp? The rear foot must leave the ground after, or at least simultaneous with, the front foot touching the ground, as observed by all free-float observers. Use the following outline to derive the consequences of this definition for the maximum speed of walking.

a Consider the three possible relationships between events FrontDown and RearUp: timelike, lightlike, and spacelike. For each of these three relationships, write down answers to the following three questions:

(1) Will the temporal order of the two events be the same for all observers?

(2) Does this relationship adequately satisfy the frame-independent definition of walking?

(3) If so, does this relationship give the maximum possible speed for walking?

Show that you answer "yes" to all three questions only for a lightlike relationship between the two events.

b A lightlike relationship between events FrontDown and RearUp means that light can just travel from one event to the other with no time left over. Let the distance between these events — the length of one step in the Earth frame — be the unit of distance and time. Show that for the limiting speed in this frame, each foot spends two units of time moving forward, then waits one unit while the light signal propagates to the other foot, then waits three units while the other foot goes through the same process. Summary: Out of six units of time, each foot moves forward at (nearly) the speed of light for two units. What is the average speed of each foot, and therefore the speed of the walker, as measured in the Earth frame?

c Draw a spacetime diagram for the Earth frame, showing worldlines for each of the robot's feet and worldlines for the connecting light flashes. Add a worldline showing the averaged motion of the torso, always located halfway between the two feet in the Earth frame. Demonstrate that this torso moves at the speed of the walker reckoned above.

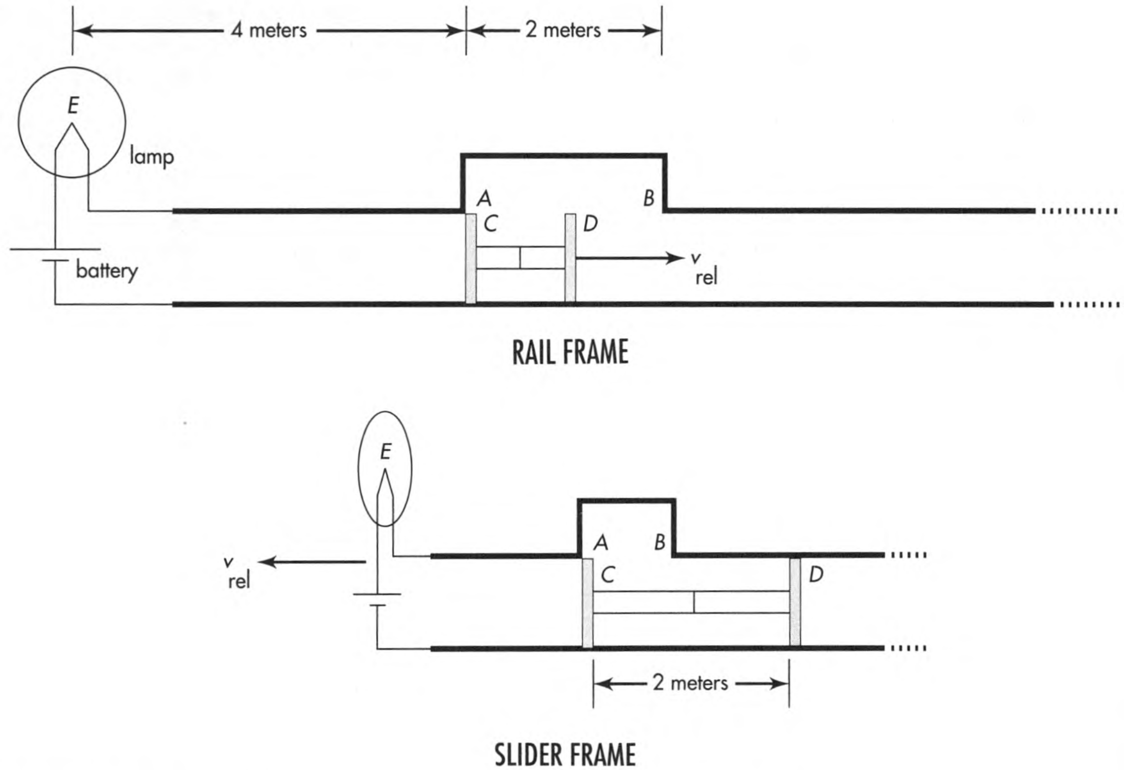
d Paul Horwitz says, "We determined the value of a maximum walking speed by finding a frame-independent definition of *walking*. Therefore this walking robot moves at the same speed as observed in every frame." Is Paul right?

Reference: George B. Rybicki, *American Journal of Physics*, Volume 59, pages 368–369 (April 1991).

6-7 the flickering bulb paradox: a project

Note: The following is too long for a regular exercise, but it has many insights worth pursuing as a longer activity. Therefore we call it a project.

Two long parallel conducting rails are open at one end but connected electrically at the other end



EXERCISE 6-7. ***Rail frame:*** *Configuration at* $t = 0$ *in the rest frame of the rails. Slider* CD *moves to the right with speed* v_{rel} *such that the Lorentz-contraction factor equals 2. The vertical legs of the slider are conductors; the horizontal crosspiece is an insulator.* ***Slider frame:*** *Configuration at* $t' = 0$ *in the rest frame of the slider. The rails and lamp move to the left with speed* v_{rel} *such that the Lorentz-contraction factor is 2.*

through a lamp and battery, as shown in the figure (rail frame). One of the rails has a square vertical offset 2 meters long. Between the rails moves (without friction) an H-shaped slider, whose vertical legs are conductors but whose horizontal crosspiece is an insulator. (Assume that the vertical legs are not perfect conductors so that, with a sufficiently powerful battery, a voltage is maintained between the rails even when they are connected by the vertical legs of the slider.) If either vertical leg of the slider connects the two rails, the electrical circuit is completed, permitting the lamp to light.

The rest (proper) length of the slider is also 2 meters, but it moves at such a speed that its Lorentz-contracted length is 1 meter in the rail frame. Hence in the rail frame there is a lapse of time during which neither leg of the slider is in contact with the upper rail. Since the circuit is open during this period, the bulb should switch off for a time and then on again — it should flicker.

The figure (slider frame) shows the configuration at $t' = 0$ in the slider frame. In this frame the slider is at rest, its length is equal to its rest length, 2 meters, while the rails, the lamp, and the battery all move to the left with a speed such that their lengths along the direction of motion are reduced by a factor of 2. In particular the offset in the upper rail is Lorentz-contracted to a length of one meter. Therefore, in the slider frame, one or the other of the slider conductors always spans the rails, so the circuit is never broken and the bulb should never switch off — it should NOT flicker!

Those trying to disprove relativity shout, "Paradox! In the rest frame of the rails the lamp switches off and then on again — it flickers. In contrast, in the rest frame of the slider the lamp stays on — it does not flicker. Yet all observers must agree: The lamp either flickers or it does not flicker. Relativity must be wrong!"

Analyze the system in sufficient detail either to demonstrate conclusively the correctness of this objection or to pinpoint its error.

Reference: G. P. Sastry, *American Journal of Physics,* Volume 55, pages 943–946 (October 1987).

6-8 the contracting spaceship paradox: a project

Note: The following is too long for a regular exercise, but it has many insights worth pursuing as a longer activity. Therefore we call it a project.

Kerwin Warnick writes in with the following paradox. A spaceship of proper length L_o accelerates from rest. Its front end travels a distance x_F in time t_F to a final speed at which the ship is contracted to half its rest length. In the same time t_F the rear end moves the same distance x_F as the front end plus the distance $L_o/2$ by which the ship has contracted. Distance traveled by the rear end $x_F + (L_o/2)$ in time t_F means an average speed $[x_F + (L_o/2)]/t_F$. Since the proper length L_o can be arbitrarily large, this average speed can be arbitrarily great, even greater than the speed of light. "This disproves relativity!" he exclaims.

Analyze this thought experiment in sufficient detail either to demonstrate conclusively the correctness of Warnick's objection or to pinpoint its error.

Reference: Edwin F. Taylor and A. P. French, *American Journal of Physics,* Volume 51, pages 889–893 (October 1983).