CHAPTER 9

GRAVITY: CURVED SPACETIME IN ACTION

9.1 GRAVITY IN BRIEF

the mutual grip of mass and spacetime

Gravity, as we see it today, does not count as a foreign force transmitted *through* space and time. Gravity manifests the curvature *of* spacetime.

Ten years after his special relativity, Einstein gave us his 1915 battle-tested and still standard theory of gravitation. Its message comes in a single simple sentence: *Spacetime grips mass, telling it how to move; and mass grips spacetime, telling it how to curve.*

Spacetime tells mass how to move

The grip of spacetime on mass enforces a central principle of special relativity: conservation of energy and momentum in a smash (Figure 9-1). The coupling of mass and spacetime geometry, far from being the weakest force in nature, is the strongest.

Mass to spacetime: "Curve!"

Now for the back-reaction, the grip mass exerts on spacetime! What curvature does that grip impose on spacetime? And how does that curvature give an account of gravity unrivaled for scope and accuracy? ◆

9.2 GALILEO, NEWTON, AND EINSTEIN

> *Only historical judgment liberates the spirit from the pressure of the past; it maintains its neutrality and seeks only to furnish light.*
> —Benedetto Croce

Galileo and Newton viewed motion as properly described with respect to a rigid Euclidean reference frame that extends through all space and endures for all time. This

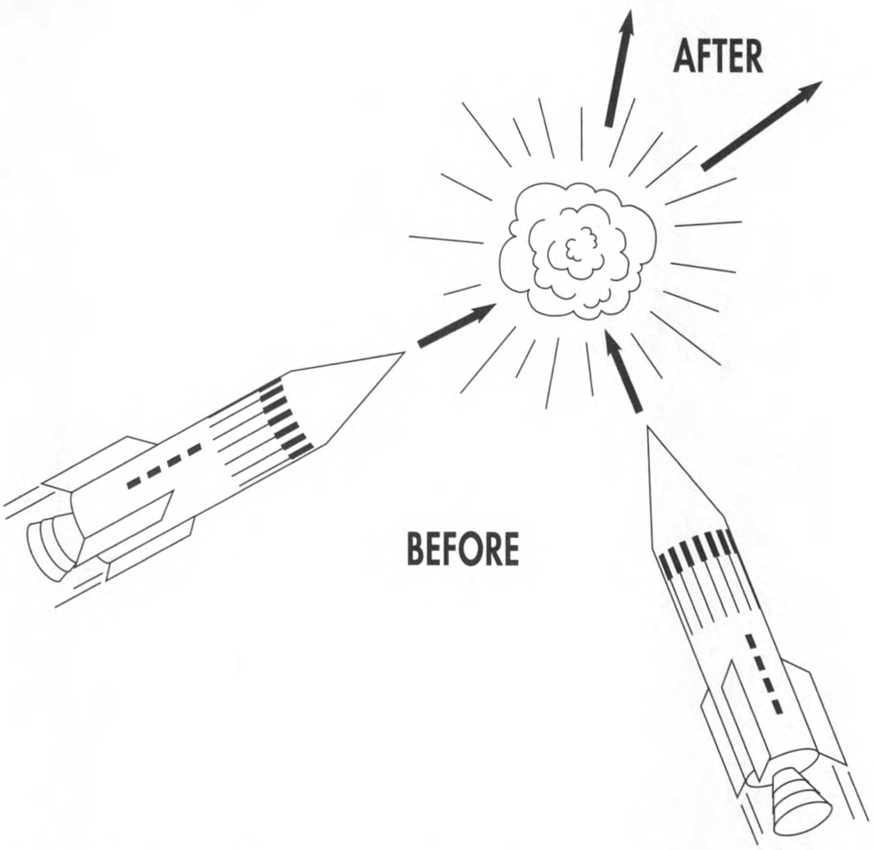

FIGURE 9-1. *Spacetime grips mass, keeping an object moving straight when free. By its power, it enforces conservation of energy and momentum in a smash.*

Newton: One global frame.
Einstein: Many local frames.

supposed reference frame stands high above the battles of matter and energy. Within this ideal space of Galileo and Newton there acts a mysterious force of gravity, an interloper from the world of physics, a foreign influence not described by geometry.

In contrast, Einstein says that there exists no mysterious "force of gravity," only the structure of spacetime itself. Climb into an unpowered spaceship, he says, and see for yourself that there is no gravity there. Physics is locally gravity-free (Chapter 2). Every free particle moves in a straight line at uniform speed. In a free-float (inertial) frame, physics looks simple. But such a frame rates as free-float in only a limited region of spacetime (Section 2.3) — a fact emphasized here by repeated use of the word "local" in describing a free-float frame.

Complications arise in describing the relation between (1) the direction of motion of a particle in one local frame and (2) the direction of motion of the same particle as observed from a nearby local frame. Any difference between the two directions is described in terms of the "curvature of spacetime," Einstein tells us. The existence of this curvature destroys the possibility of describing motion with respect to a single ideal Euclidean reference frame that pervades all space. What is simple is the geometry in a region small enough to look flat.

How did the views of Galileo, Newton, and Einstein develop? And what is the concrete substance of the strange phrase "curvature of spacetime"?

9.3 LOCAL MOVING ORDERS FOR MASS

moving orders from the local commander, spacetime!

Navigation satellites near Earth drift away from "perfect" orbits because thin air and solar radiation pressure affect their motion. Figure 9-2 shows an experimental satellite that carries a "conscience" designed to assure that the same motion will be maintained

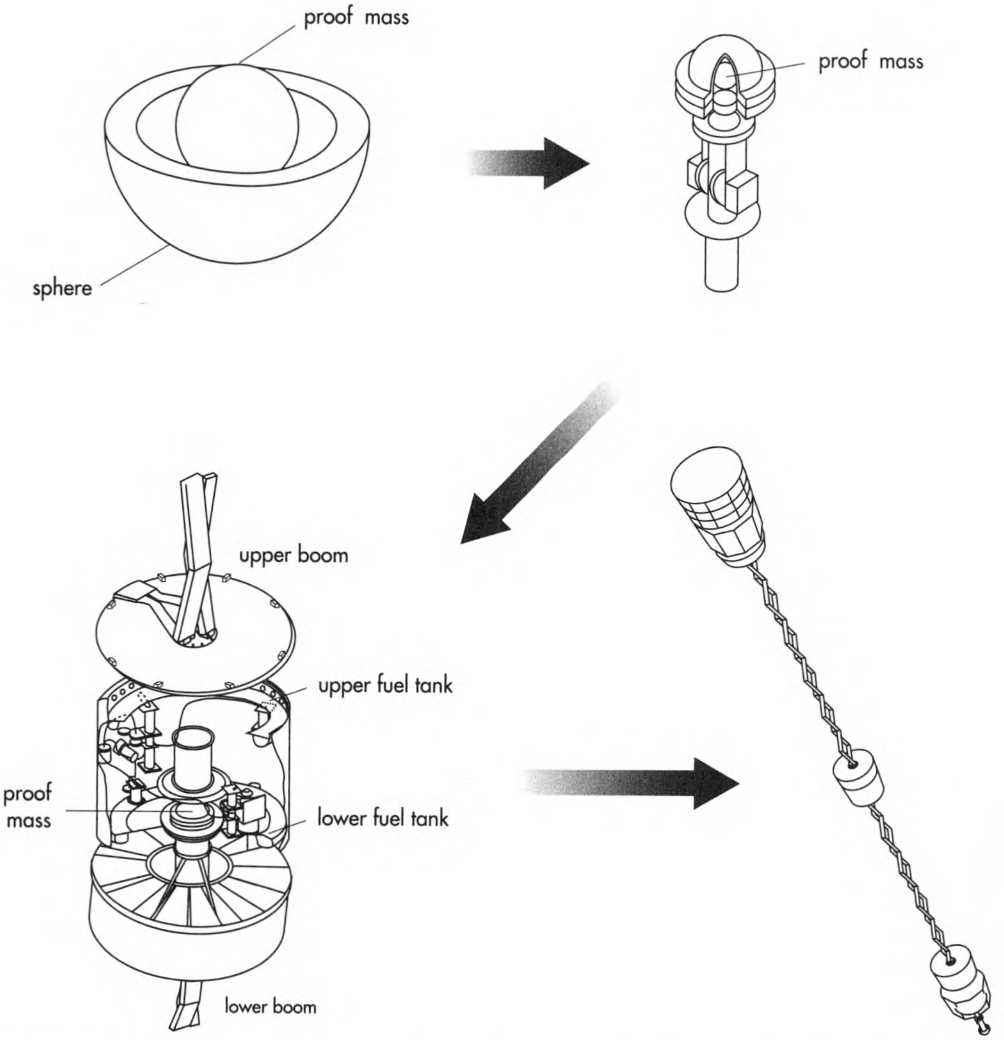

FIGURE 9-2. ***"Conscience-guided" satellite.*** *A satellite in orbit around Earth is subject to small accelerations due to solar radiation pressure and residual atmospheric drag. Uncorrected, these accelerations are between* $10^{-6}g$ *and* $10^{-9}g$*, where g is the acceleration of gravity at Earth's surface. The acceleration was reduced to* $5 \times 10^{-12}g$ *for more than a year in orbit by use of a conscience or proof mass and the Disturbance Compensation System (DISCOS) mounted on a TRIAD U.S. Navy satellite. The conscience, a gold–platinum sphere 2.2 centimeters in diameter, floats freely inside a spherical housing. Any nongravitational force results in an incremental velocity change. The floating proof mass continues in its original state of motion in an ideal friction-free environment. Observing the proof mass through capacitor sensing devices, the satellite becomes aware that it is not keeping up with the motion demanded by the proof mass. An opposite vernier rocket fires long enough to bring the spaceship back into concord with its proof mass—its conscience. To reduce gravitational effects of the satellite itself on the proof mass, fuel for the vernier rockets is stored in donut-shaped tanks placed symmetrically above and below the proof mass; power supply and radio transmitter are each held at the end of a boom 2.7 meters long on either side of the control unit. For an Earth-based microgravity environment, recall Figure 2-3. (Used with permission of AIAA. Journal of Spacecraft.)*

ISAAC NEWTON

Woolsthorpe, December 25, 1642—Kensington (London), March 20, 1727

"The marble index of a mind forever
Voyaging through strange seas of thought, alone."—*Wordsworth*

★ ★ ★

"I do not know what I may appear to the world; but to myself I seem to have been only like a boy, playing on the sea-shore, and diverting myself, in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."—*Newton*

★ ★ ★

"Why do I call him a magician? Because he looked on the whole universe and all that is in it as a *riddle*, as a secret which could be read by applying thought to certain evidence, certain mystic clues which God had laid about the world to allow a sort of philosopher's treasure hunt to the esoteric brotherhood. He believed that these clues were to be found partly in the evidence of the heavens and in the constitution of elements (and that is what gives the false suggestion of his being an experimental natural philosopher), but also partly in certain papers and traditions handed down by the brethren in an unbroken chain back to the original cryptic revelation in Babylonia. He regarded the universe as a cryptogram set by the Almighty—just as he himself wrapt the discovery of the calculus in a cryptogram when he communicated with Leibnitz. By pure thought, by concentration of mind, the riddle, he believed, would be revealed to the initiate."—*Keynes*†

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when it encounters these disturbances as when it moves through perfect emptiness. The "conscience" — called a **proof mass** — is a separate sphere that floats inside the larger ship. The proof mass undergoes no acceleration relative to the ship as long as the ship moves freely. When relative motion does occur, the error in the tracking must be due to the satellite. By small rockets the satellite gives itself a brief spurt of acceleration and comes back into step with the inner proof mass — the satellite's conscience. Though resistance is present, the rocket thrust overcomes it. The satellite takes the same course it would have taken had both resistance and thrust been absent.

"Conscience-guided" satellite. What guides the conscience?

As satellite and proof mass come to empty space, they fly through it in perfect step, without use of rockets or sensing devices. What a remarkable harmony they present! The inner proof mass does not see outer space. It does not touch, feel, or see the ship that surrounds it on every side. Yet it faithfully tracks the ship's route through spacetime. Moreover, this tracking is as perfect when the proof mass is made of aluminum as when it is made of gold. How do proof masses — of whatever atomic constitution and whatever construction — know enough to follow a standard worldline? Where does mass get its moving orders?

Locally, answers Einstein. From a distance, answers Newton.

Einstein says that the proof mass gets its information in the simplest way possible. It responds to the structure of spacetime in its immediate vicinity. It moves on a straight line in the local free-float frame. No simpler motion and no straighter motion can be imagined.

Newton says that the inner proof mass gets its information about how to move from a distance, via a "force of gravity." Motion relative to what? Motion relative to an ideal, God-given, never-changing Euclidean reference frame that spans all of space and endures for all time. He tells us that the proof mass would have moved along an ideal straight line in this global frame had not Earth deflected it. How can this ideal line be seen? How sad! There is nothing, absolutely nothing, that ever moves along this ideal line. It is an entirely imaginary line. But it nevertheless has a simple status, Newton tells us, in this respect: Every satellite and every proof mass, going at whatever speed, is deflected away from this ideal line at the same acceleration (Figure 9-3).

Physics is simple only when analyzed locally

Einstein says: Face it; there is no ideal background Euclidean reference frame that extends over all space. And why say there is, when even according to Newton no particle, not even a light ray, ever moves along a straight line in that ideal reference frame. Why say spacetime is Euclidean on a large scale when no evidence directly supports that hypothesis? To try to set up an all-encompassing Euclidean reference frame and attempt to refer motion to it is the wrong way to do physics. Don't try to describe motion relative to faraway objects. *Physics is simple only when analyzed locally.* And locally the worldline that a satellite follows is already as straight as any worldline can be. Forget all this talk about "deflection" and "force of gravitation." I'm inside a spaceship. Or I'm floating outside and near it. Do I feel any "force of gravitation?" Not at all. Does the spaceship "feel" such a force? No. Then why talk about it? Recognize that the spaceship and I are traversing a region of spacetime free of all force. Acknowledge that the motion through that region is already ideally straight.

How can one display the straightness of the motion? Set up a local lattice of meter sticks and clocks, a local free-float (inertial) reference frame — also called a Lorentz reference frame (Chapter 2). How does one know the frame is free-float? Watch every particle, check every light ray, test that they all move in straight lines at uniform speed relative to this frame. And having thus verified that the frame is free-float, note that the proof mass too moves at a constant speed in a straight line — or remains at rest — relative to this local free-float frame. What could be simpler than the moving orders for mass: "Follow a straight line in the local free-float reference frame." Does a proof mass have to know the location of Earth and Moon and Sun before it knows how to move? Not at all! Surrounded on all sides by the black walls of a satellite, it has only to sense the local structure of spacetime — right where it is — in order to follow the correct track.

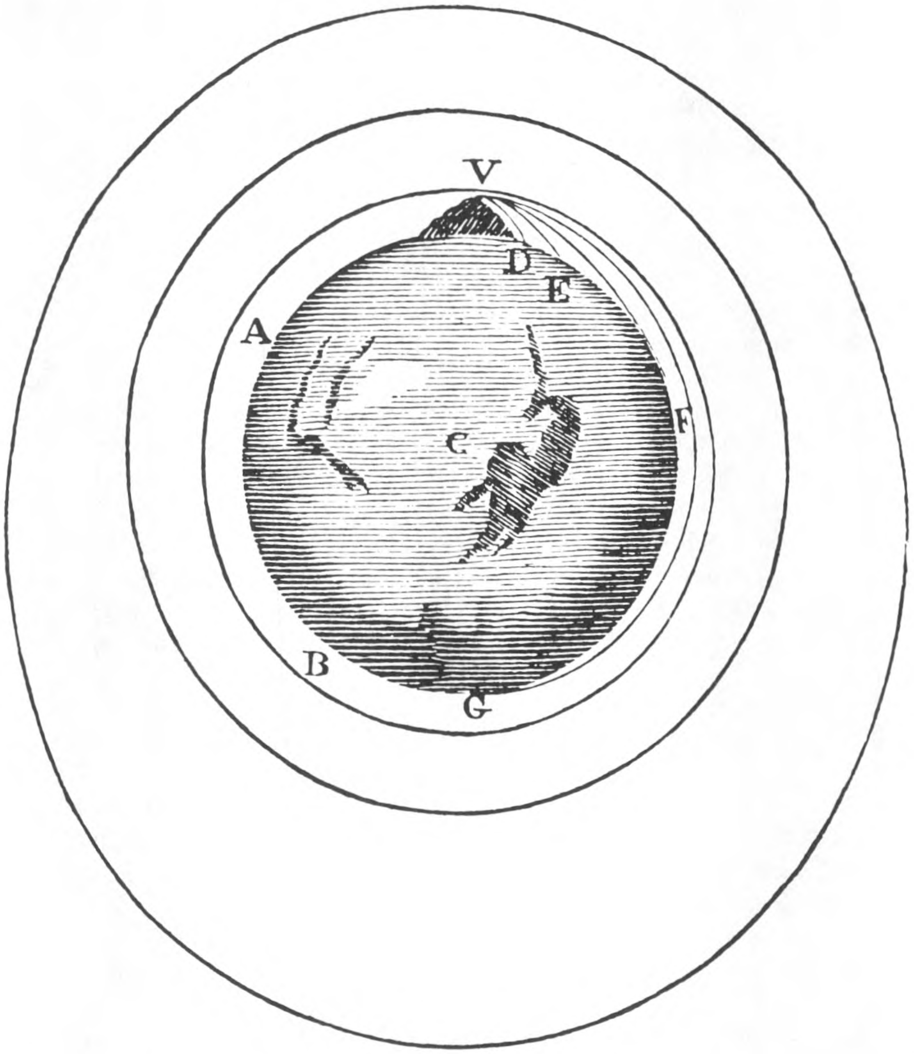


FIGURE 9-3. *In Newtonian mechanics different particles going at different speeds are all deflected away from the ideal straight line with equal acceleration. In this respect there is no difference in principle between the fall of a projectile and the motion of a satellite. In this picture of Newton's published in 1686, cannon of successively greater power mounted on a mountaintop fire out their balls horizontally. The more powerful cannon launches a satellite. The outer two curves show other possible satellite orbits. In brief, Newton has one global reference frame, but within this reference frame no satellite is ever gravity-free, and no particle ever moves in a straight line at constant speed. Einstein, in contrast, makes use of many local regions in each of which the geometry is Lorentzian (as in special relativity); the laws of gravitation arise from the lack of ideality in the relation between one local region and the next (gravitation; spacetime curvature; general relativity).*

9.4 SPACETIME CURVATURE

not one but two particles witness to gravitation

Splendid! And also simple! But isn't Einstein's view of motion *too* simple? We started out interested in the motion of a spaceship around Earth and in "gravitation." We seem to have ended up talking only about the motion of the satellite — or the proof mass — relative to a strictly local inertial reference frame, a trivially simple straight-line motion. Where is there any evidence of "gravitation" to be seen in that? Nowhere.

This is the great lesson of Einstein: Spacetime is always and everywhere locally Lorentzian. No evidence of gravitation whatsoever is to be seen by following the motion of a single particle in a free-float frame.

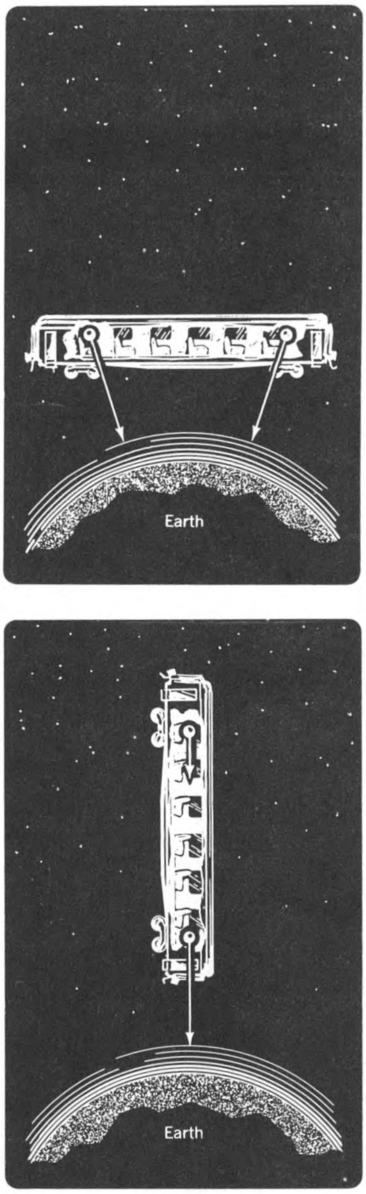
One has to observe the relative acceleration of two particles slightly separated from each other to have any proper measure of a gravitational effect. Separated by how much? That depends on the region of spacetime and the sensitivity of the measuring equipment. Two ball bearings with a *horizontal* separation of 20 meters, dropped from a height of 315 meters above Earth's surface with 0 initial relative velocity, hit the ground 8 seconds later (24×10^8 meters of light-travel time later) with a separation that has been *reduced* by 10^{-3} meter (Section 2.3). Two ball bearings with a *vertical* separation of 20 meters, dropped from a height of 315 meters with 0 initial relative velocity, in the same 8 seconds *increase* their separation by 2×10^{-3} meter. To measuring equipment unable to detect such small relative displacements the ball bearings count as moving in one and the same free-float reference frame. No evidence for gravitation is to be seen. More sensitive apparatus detects the **tide-producing action** of gravity — the accelerated shortening of horizontal separations parallel to Earth's surface, the accelerated lengthening of vertical separations. Each tiny ball bearing still moves in a straight line in its own local free-float reference frame. But now — with the new precision — the region of validity of the one free-float reference frame does not reach out far enough to give a proper account of the motion of the other steel ball. The millimeter or two discrepancy is the way "gravity" manifests itself.

Tidal acceleration displays gravity as a local phenomenon. No mention here of the distance of the steel balls from the center of Earth! No mention here of acceleration relative to that center! The only accelerations that come into consideration are those of nearby particles relative to each other, the tidal accelerations described in the preceding paragraph.

These relative accelerations double when the separations are doubled. The true measure of the tide-producing effect has therefore the character of an acceleration per unit of separation. Let the acceleration be measured in meters of distance per meter of light-travel time per meter of light-travel time; that is, in units meters/meter^2 or 1/meter [$x = (1/2)at^2$, so $a = 2x/t^2$]. Then the measure of the tide-producing effect (different for different directions) has the units (acceleration/distance) or (1/meter^2). In the example, in the two horizontal directions this quantity has the value [$2(-0.001$ meter$)/(24 \times 10^8$ meter$)^2$]/20 meter $= -17.36 \times 10^{-24}$ meter^{-2} and in the vertical direction twice the value and the opposite sign: $+34.72 \times 10^{-24}$ meter^{-2}. The tide-producing effect is small but it is real and it is observable. Further, it is a locally defined quantity. And Einstein tells us that we must focus our attention on locally defined quantities if we want a simple description of nature.

Einstein says more: This tide-producing effect does not require for its explanation some mysterious force of gravitation, propagated through spacetime and additional to the structure of spacetime. Instead, it can and should be described in terms of the geometry of spacetime itself as the **curvature of spacetime.**

Though Einstein speaks of four-dimensional spacetime, his concepts of curvature can be illustrated in terms of two-dimensional geometry on the surface of a sphere.



Einstein's railway coach in free fall.

9.5 PARABLE OF THE TWO TRAVELERS

space curvature on a sphere accounts for relative acceleration of travelers

One traveler, A, stands at the equator, ready to travel straight north. A's companion B, standing against him shoulder to shoulder, wheels 90 degrees and marches straight

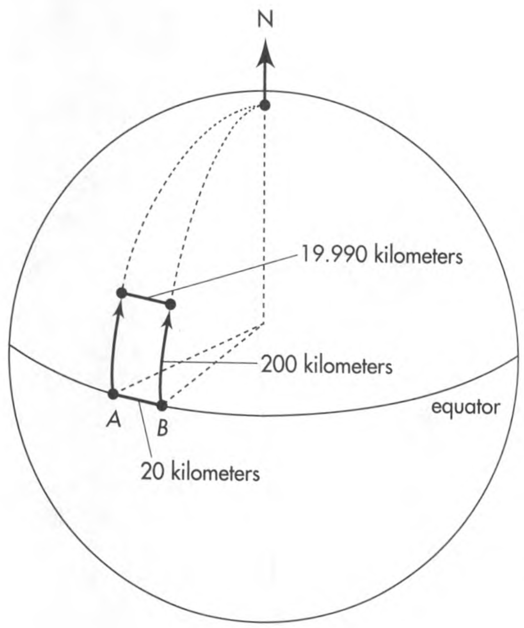


FIGURE 9-4. *Travelers* A *and* B, *starting out parallel and deviating neither to the left nor to the right, nevertheless find themselves approaching each other after they have traveled some distance. Interpretation 1: Some mysterious force of "gravitation" is at work. Interpretation 2: They are traveling on a curved surface. Figure not drawn to scale.*

east. She paces off 20 kilometers along the equator. There she again turns a sharp 90 degrees and faces straight north. Both travelers now start north and travel 200 kilometers (Figure 9-4). In the beginning their tracks are strictly parallel. Moreover, no travelers could be more conscientious than they are in continuing precisely in their original directions. Each of them deviates neither to the right nor to the left. Yet an umpire sent out to measure their separation after their 200-kilometer treks finds it to be less than the original 20 kilometers. Why? We know perfectly well: The surface of the globe is curved. If they continue north, their paths will meet at the north pole.

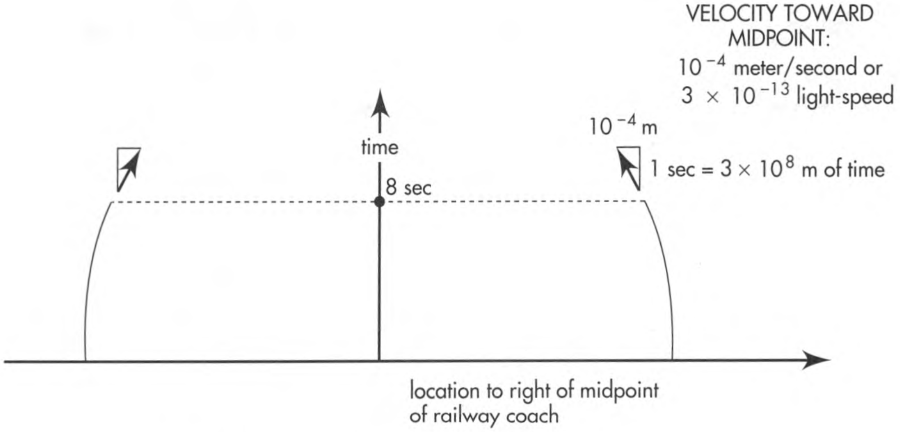
Already at this early stage of their trip the travelers are approaching each other, although they had started out not approaching at all. Initially their velocity relative to one another was zero; now they move toward one another with a small relative velocity. In this sense they are slowly accelerating toward each other.

Curvature of Earth demonstrated by change in separation of two originally parallel paths

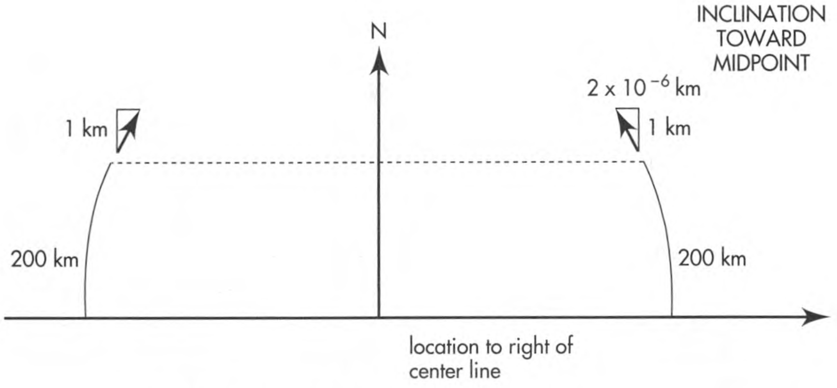
The travelers accelerate toward each other as surely as two tiny ball bearings in a free-fall horizontal railway coach accelerate toward each other (Figure 9-5). We ascribe the relative acceleration of ball bearings in the railway coach to the "tidal" effects of nonuniform gravitation near Earth. To be sure, the relevant picture for the travelers is the two-dimensional curved space of the surface of Earth, whereas what counts for the ball bearings is curvature of spacetime. This parallelism between the geometrical concept of curvature and the gravitational concept of tide-producing effect foreshadows Einstein's geometrical interpretation of gravity.

The two travelers, who started out so conscientiously on parallel tracks and deviated neither to the left nor to the right, have been told by the umpire of distances that despite all precautions they are now slowly accelerating toward one another. They blame this development on the existence of some mysterious "gravitational force" that deflects their paths. They explore the nature of this "gravitational force." Repeating the travel with bicycles, motorcycles, light cars, and heavy trucks all moving northward with the same speed, they find always the same relative acceleration toward one another. They conclude that the "gravitational force" leads to the same acceleration of all objects, no matter what they are made of or how massive they are.

Learned would-be pundits analyze the motion of travelers. They say, in words utterly mysterious to us, "See here. You find the same acceleration for every vehicle



BALL BEARINGS FALLING "DOWN" IN RAILWAY COACH



TRAVELERS HEADED "NORTH" ON EARTH

FIGURE 9-5. ***Comparison of the paths of northward travelers on Earth's surface with the worldlines of ball bearings released side by side from rest near Earth's surface.*** *In both cases the "path" of each "traveler" starts parallel with that of the second traveler (zero initial relative velocity). In both cases this "path" gradually inclines toward the centerline ("relative acceleration"). In both cases the paths can be accounted for in terms of the local curvature of geometry (curvature of Earth's surface for the travelers; curvature of spacetime geometry—gravitation!—for the ball bearings). In each diagram, vertical distances are drawn—for vividness—to a different scale than horizontal distances. Both diagrams suffer from this additional imperfection: they attempt to show, on the flat Euclidean surface of this page, trajectories that can be correctly represented only in terms of a curved geometry.*

you try. This means that the ratio of gravitational mass to inertial mass is the same for all sorts of objects. You have made a great discovery about mass."

Curvature alone accounts for relative acceleration

All this time we and our space-traveler friends are looking down from on high. We see the many treks. We watch the many measurements of distance. Through our intercommunication system we hear and approve as our friends on the ground interpret distance shortening as relative acceleration—and relative acceleration as "gravitation." But then they get into weighty discussions. They start speaking of "gravitation" as action at a distance. We smile. What is at issue—we know—is not action at a distance at all, but the geometry of curved space. All this talk about the identity of "gravitational mass" and "inertial mass" completely obscures the truth. Curvature and nothing more is all that is required to describe the increasing rate at which A and B approach each other.

9.6 GRAVITATION AS CURVATURE OF SPACETIME

spacetime curvature accounts for tidal accelerations of objects

Spacetime curvature demonstrated by change in separation of two originally parallel worldlines

Einstein smiles, too, as he hears gravitation described as action at a distance. Curvature of spacetime and nothing more, he tells us, is all that is required to describe the millimeter or two change in separation in 8 seconds of two ball bearings, originally 20 meters apart in space above Earth, and endowed at the start with zero relative velocity. Moreover, this curvature completely accounts for gravitation.

"What a preposterous claim!" is one's first reaction. "How can such minor — and slow — changes in the distance between one tiny ball and another offer any kind of understanding of the enormous velocity with which a falling mass hits Earth?" The answer is simple: Many local reference frames, fitted together, make up the global structure of spacetime. Each local Lorentz frame can be regarded as having one of the ball bearings at its center. The ball bearings all simultaneously approach their neighbors (curvature). Then the large-scale structure of spacetime bends and pulls nearer to Earth (Figure 9-6). In this way many local manifestations of curvature add up to give the appearance of long-range gravitation originating from Earth as a whole.

Acceleration toward Earth: Totalized effect of relative accelerations, each particle toward its neighbor, in a chain of test particles that girdles globe

In brief, the geometry used to describe motion in any local free-float frame is the flat-spacetime geometry of Lorentz (special relativity). Relative to such a local free-float frame, every nearby electrically neutral test particle moves in a straight line with constant velocity. Slightly more remote particles are detected as slowly changing their velocities, or the directions of their worldlines in spacetime. These changes are described as tidal effects of gravitation. They are understood as originating in the local curvature of spacetime.

From the point of view of the student of local physics, gravitation shows itself not at all in the motion of one test particle but only in the change of separation of two or more nearby test particles. "Rather than have one global frame with gravitational forces we have many local frames without gravitational forces." However, these local dimension changes add up to an effect on the global spacetime structure that one interprets as "gravitation" in its everyday manifestations.

In contrast, Newton supposed the existence of one ideal overall reference frame. For him, "Absolute space, in its own nature, without relation to anything external, remains always similar and immovable." The ball bearing or spaceship is regarded by Newton as actually accelerated with respect to this ideal frame. The "gravitational force" that accelerates it acts mysteriously across space and is produced by distant objects. That the man in the spaceship finds no evidence either of the acceleration or the force is an accident of nature, according to the Newtonian view. Pundits used to interpret this accident of nature as the fortuitous equality of "gravitational mass" and "inertial mass" or in other "learned" ways.

In conversations with one of the authors of this book at various times over the years, Einstein emphasized his great respect for Newton and, in particular, his admiration for Newton's courage and judgment. He stressed that Newton was even better aware than his seventeenth-century critics of the difficulties with the ideas of absolute space and time. To postulate those ideas was nevertheless the only practical way to get on with the task of describing motion in Newton's century. In effect, Newton chopped the problem of motion into two parts: (1) space and time and their meaning: ideas that were puzzling but usable and that were destined to be clarified only 230 years later and (2) the laws of acceleration with respect to that idealized spacetime: laws that Newton gave the world.

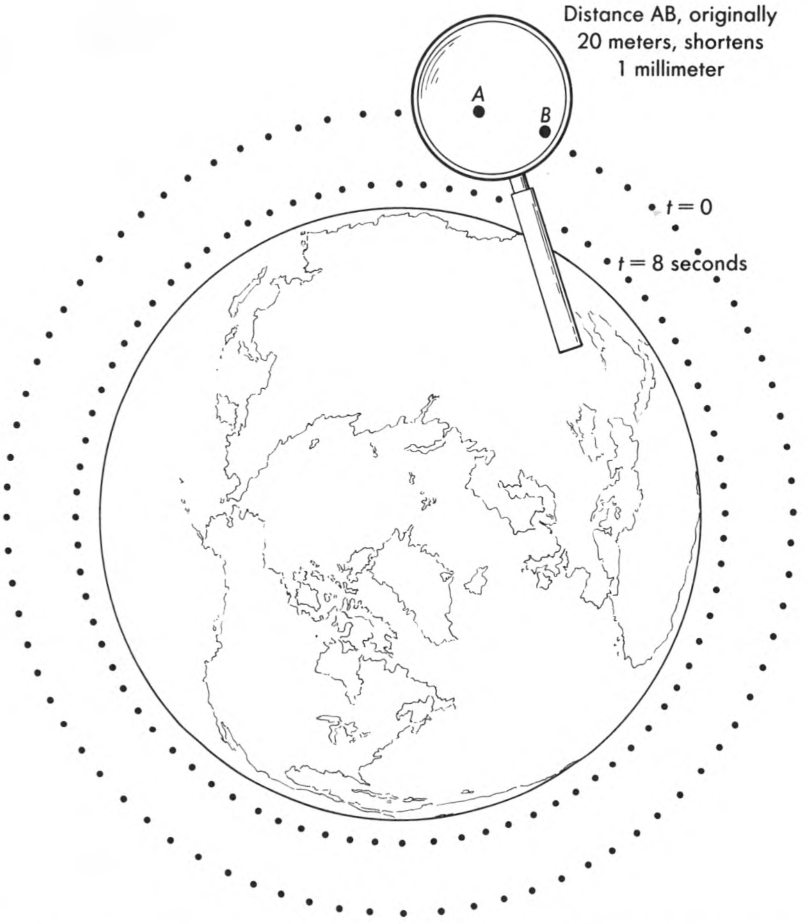


FIGURE 9-6. ***Local curvature adding up to the appearance of long-range gravitation.*** *The shortening of distance between any one pair,* A *and* B, *of ball bearings is small when the distance itself is small. However, small separation between each ball bearing and its partner demands many pairs to encompass Earth. The totalized shortening of the circumference in any given time — the shortening of one separation times the number of separations — is independent of the fineness of the subdivision. That totalized pulling in of the circumference carries the whole necklace of masses inward. This is free fall, this is gravity, this is a large scale motion interpreted as a consequence of local curvature. Example:*

Original separation between A *and* B *— and every other pair: 20 meters*
Time of observation: 8 seconds
Shortening of separation in that time: 1 millimeter
Fractional shortening: $1 \text{ millimeter}/20 \text{ meters} = 1/20{,}000$
Circumference of Earth (length of airy necklace of ball bearings): 4.0030×10^7 *meters*
Shrinkage of this circumference in 8 seconds: $1/20{,}000 \times 4.0030 \times 10^7 \text{ meters} = 2001.5 \text{ meters}$
Decrease in the distance from the center of Earth (drops by the same factor $1/20{,}000$*):*
$1/20{,}000 \times 6.371 \times 10^6 \text{ meters} = 315 \text{ meters}.$

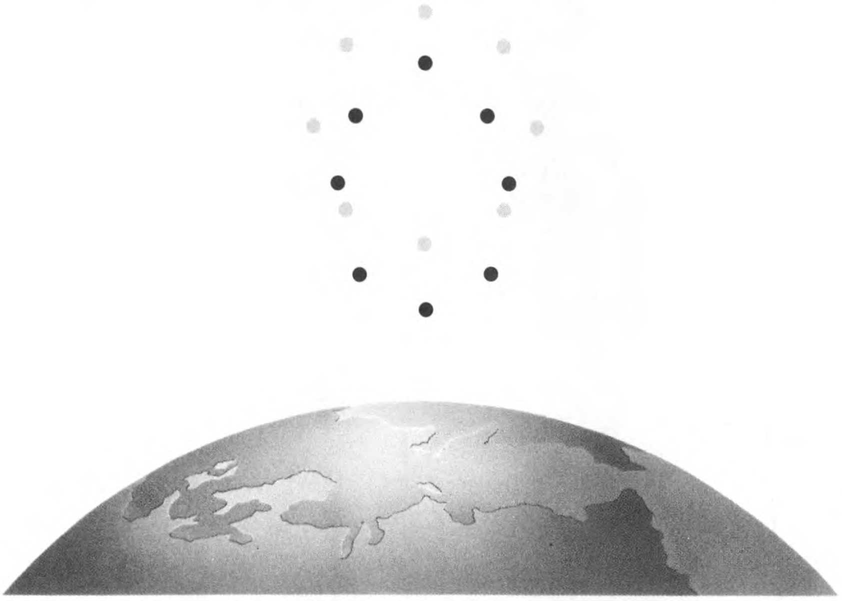
This apparently large-scale effect is caused — in Einstein's picture — by the addition of a multitude of small-scale effects: the changes in the local dimensions associated with the curvature of geometry (failure of B *to remain at rest as observed in the free-float frame associated with* A*).*

What is the source of the curvature of spacetime? Momenergy is the source. In Chapter 8 we saw the primacy of momenergy in governing interactions between particles. Crash of mass on mass, no matter how elastic or how destructive, leaves the total momenergy of the system quite unaltered. By what miracle does this come about? Education of momenergy from birth onward to good behavior? Goodness of heart?

BOX 9-1

HOW SPACETIME CURVATURE CARRIES INFLUENCE FROM ONE MASS TO ANOTHER

The necklace of ball bearings (Figure 9-6) as they approach Earth, examined more closely, reveals a remarkable feature of spacetime curvature outside a great, essentially uniform, essentially isolated sphere of mass. The curvature in its character is totally "tide-producing," totally "noncontractile."



An array of test masses covering the surface of a hollow sphere freely floating above the Earth's surface will shrink in two dimensions and lengthen in one. The volume remains constant; only the shape changes. This change is evidence of the noncontractile, tide-driving spacetime curvature outside Earth.

What do these descriptive terms mean, and how do we verify that they apply? We look at a cluster of ball bearings dotted here and there over the surface of an imaginary small sphere, all momentarily at rest relative to each other and relative to Earth. That shape, however, as the seconds tick by, changes from sphere to ellipsoid. How come? First let's look at the two dimensions of the sphere that lie perpendicular to each other but parallel to Earth's surface. Both these dimensions of the sphere shrink as the ball bearings converge toward Earth's center. The up-down dimension of the pattern, however, lengthens, and twice as much. Why? Newton says because of the greater gravitational acceleration of the one nearer Earth. Einstein says because two-percent stretch in that dimension compensates one-percent shrinkage in the other two dimensions and keeps the volume of the pattern unchanged. Spacetime curvature, yes; but a totally noncontractile curvature. Einstein's famous equation, stated in simple terms, tells us how spacetime curvature responds to mass:

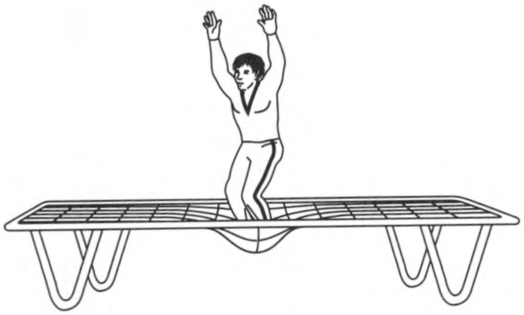
$$\begin{pmatrix}\text{appropriate measure of}\\ \text{spacetime contractile curvature}\\ \text{at any place, any time,}\\ \text{in any Lorentz frame}\end{pmatrix} = \begin{pmatrix}\text{a universal}\\ \text{constant}\end{pmatrix} \times \begin{pmatrix}\text{density of energy}\\ \text{at that locale}\\ \text{perceived in that}\\ \text{Lorentz frame}\end{pmatrix}$$

Outside, no mass, no energy, a spacetime curvature that is totally noncontractile. Inside Earth, however, there is mass, therefore there is energy—or in a mov-

Spacetime controls momenergy

Obedience to the eyes of a corps of bookkeepers? No, Einstein taught us. The enforcing agency does not lie far away. It's close at hand. It's the geometry of spacetime, right where the crash takes place. Not only does spacetime grip isolated mass, telling it how to move. In addition, in a crash it sees to it that the participants neither gain nor lose momenergy. But there is more! Spacetime, in so acting, cannot ing frame, energy plus energy flow — and therefore spacetime curvature there has a contractile character. The ball bearings — when shafts are drilled for them so that not one of them encounters any obstacle to free-float motion — start to converge vertically as well as horizontally. The volume shrinks. That, overlooking details, is what we mean when we say that "mass grips spacetime, telling it how to curve."

There is no Earth mass out at Moon's orbit. How then does Einstein's spacetime geometry account for Moon's motion? Answer: Earth's mass imposes on spacetime a contractile curvature throughout Earth's interior, as a jumper's feet impose a contractile curvature on a trampoline. That contractile curvature, where the feet push, forces on the surrounding nontear fabric a corresponding lateral stretch. That effect transmits itself in ever more dilute measure to the ever more remote regions of the trampoline.



The deformation of the nontear trampoline fabric under the jumper's feet and elsewhere is analogous to the nontear curvature of spacetime geometry inside Earth and elsewhere.

Likewise spacetime does not tear. Its fabric just above Earth's surface experiences the same lateral contractility as it does just below the surface. Not so with the curvature in the two-dimensional domain defined by time and by direction perpendicular to Earth's surface. In that one plane, curvature within Earth is contractile but suddenly jumps just above Earth's surface to the opposite character. Hence the tide-producing character of spacetime curvature outside Earth. A point twice as far from Earth's center lies on an imaginary Earth-centered sphere that encompasses eight times the volume. There the tide-producing curvature experiences eight times the dilution and has one eighth the strength. Despite this rapid dilution of tide-producing power with distance, it has strength enough at Moon, 60 Earth radii away from Earth's center, to deform Moon from sphere to ellipsoid, 1738.35 kilometers in radius along the Earth–Moon direction, 1738.15 kilometers in radius for each of the other two perpendicular directions.

Easy as it is to regard Earth as running the whole show, Moon too has its part. Like an infant standing on the trampoline some distance from its mother, it imposes its own small curvature on top of the curvature evoked by Earth. That additional curvature, contractile in Moon's interior, has tide-driving character outside. Were the Earth an ideal sphere covered by an ideal ocean of uniform depth, then Moon would draw that ocean's surface 35.6 centimeters higher than the average in two domains, one directly facing Moon, one directly opposite to it — simultaneously lowering those waters 17.8 centimeters below the average on the circle of points midway between the two. (These low figures show how important are funneling and resonant sloshing in determining heights of actual ocean tides on Earth.)

The local contractile curvature of spacetime at Moon's location added up along Moon's path yields the appearance of long-range gravitation, similar to that illustrated in Figure 9-6. Box 2-1 tells a little of the many influences that have to be taken into account in any fuller treatment of the tides.

maintain the perfection assumed in textbooks of old. To every action there is a corresponding reaction. *Spacetime acts on momenergy, telling it how to move; momenergy reacts back on spacetime, telling it how to curve.* This "handshake" between momenergy and spacetime is the origin of momenergy conservation — and the source of spacetime curvature that leads to gravitation (Box 9-1).

Momenergy tells spacetime how to curve

9.7 GRAVITY WAVES

gravitational energy moving at light speed

Gravity waves from collapsing matter

In the depths of an ill-fated, collapsing star, billions upon billions of tons of mass cave in and crash together. The crashing mass generates a wave in the geometry of space—a wave that rolls across a hundred thousand light-years of space to "jiggle" the distance between two mirrors in our Earthbound gravity-wave laboratory.

A cork floating all alone on the Pacific Ocean may not reveal the passage of a wave. But when a second cork is floating near it, then the passing of the wave is revealed by the fluctuating separation between the two corks. So too for the separation of the two mirrors. There is, however, this great difference. The cork-to-cork distance reveals a momentary change in the two-dimensional geometry of the surface of the ocean. The

BOX 9-2

COMPACT STELLAR OBJECTS

Three kinds of astronomical objects exist comparable in mass to Sun but very much smaller. Two of these have been observed; the third seems an inevitable result of Einstein's theory.

A **white dwarf star** is a star of about one solar mass, with radius about 5000 kilometers. (The radius of Earth is 6371 kilometers.) This gives the white dwarf a density of approximately 10^9 kilograms/meter^3 (or one metric ton per cubic centimeter). As of 1990, approximately 1500 white dwarfs have been identified.

White dwarfs were observed and studied astronomically long before they were understood theoretically. Today we have come to recognize that a white dwarf is a star that quietly used up its fuel and settled gently into this compact state. The electrons and nuclei that make up the body of a white dwarf are not separated into atoms. Instead, the electrons form a gas in which the nuclei swim. The pressure of this "cold" electron gas keeps the white dwarf from collapsing further.

S. Chandrasekhar calculated in 1930 that no white dwarf can be more massive than approximately 1.4 solar masses ("Chandrasekhar limit") without collapsing under its own gravitational attraction. His analysis assumed the mix of electrons and nuclei to be unaltered under compression by a load so heavy, an assumption that had to be modified in later years. Today we recognize that enormous compressions squeeze electrons into combining with protons to make neutrons. At compressions near the Chandrasekhar limit, the electron gas transforms into a neutron gas, the interior of the star becomes a giant nucleus, and the whole nature of the compact object changes to that of a neutron star.

A **neutron star** has roughly the same density as an atomic nucleus, of the order of 10^{17} kilograms/meter^3, or one Earth mass per cube of edge length 400 meters. The radius of a neutron star is approximately 10 kilometers.

mirror-to-mirror distance reveals a momentary change in the three-dimensional geometry of space itself.

How to detect gravity waves

The idea of extracting energy from ocean waves is old. After all, the ability of a water wave to change a distance lets itself be translated into the ability to do work. The same reasoning applies to a gravity wave. Because it can change distance, it can do work. It carries energy. Energy once resident as mass in the interior of a star has radiated out to us and to all the universe.

Of all the workings of the grip of gravity, none is more fascinating or opens up for exploration a wider realm of ideas than a gravity wave. None pushes to a higher pitch the art of detecting a small effect, and none gives more promise of providing an unsurpassable window on cataclysmic events deep inside troubled stars. Nevertheless, no other great prediction of Einstein's geometric theory of gravity stands today so far from triumphant exploitation. As of this writing, not one of the nine ingenious

How often is a neutron star formed? Towards answering this still open question we have one important lead: In our own galaxy we see one supernova explosion on average about every 300 years [most recent supernova in the Large Magellanic Cloud, a satellite structure near our galaxy, on February 23, 1987; one seen by Kepler, October 13, 1604; one seen by Tycho Brahe, November 6, 1572; earlier ones: 1181 A.D.; July 4, 1054 A.D.; 1006 A.D. (the brightest); 185 A.D.; and two possibles in 386 A.D. and 393 A.D.]. In such an event a star teetering on the edge of instability finally collapses. The Niagara Falls of infalling mass in some cases go too far and overcompress the inner region of the star. That region thereupon acts like a spring, or explosive charge, and drives off the outer portions of the star. This explains the spectacular luminosity that is such a prominent feature of a supernova. The core that remains becomes a neutron star in some events, it is believed, in others a black hole.

Neutron stars were predicted in 1934 but not observed until 1968. Many neutron stars spin rapidly—with a period as short as a few milliseconds. A neutron star typically has an immense magnetic field. When that field is aligned at an angle relative to the axis of spin of the star (as in Earth, for example), it sweeps around like a giant whisk brush through the plasma in the space around the star. The periodic shock to the electrons of the plasma from the periodic arrival of this field excites those electrons to radiate periodic pulses of radio waves and visible light—both observed on Earth. Because of this behavior, such neutron stars are called **pulsars.** As of 1990, nearly 500 pulsars have been identified.

A **black hole** is an object created when a star collapses to a size so small that strong spacetime curvature prevents it from communicating outward with the external universe. Even light cannot escape from a black hole, whence its name. No one who accepts general relativity has found any way to escape the prediction that black holes must exist in our galaxy. Strong evidence for the existence of black holes has been found, but it is not yet convincing to all astrophysicists. A black hole can have a mass as small as a few times the mass of our Sun. A black hole of three solar masses would have a "radius" of about 9 kilometers. There is no theoretical upper limit to its mass.

detectors built to this day has proved sensitive enough to secure any generally agreed detection of an arriving gravity wave.

Does any truly simple line of reasoning assure us that gravity will inescapably carry energy away from two masses that undergo rapid change in relative position? Yes is the conclusion of a little story that savors of mythology. The Atlas of our day, zooming through space in free float, insists as much as ever on maintaining physical fitness. He pumps iron, not by raising iron against the pull of Earth's gravity, but by throwing apart two identical great iron spheres, Alpha and Beta. He floats between those minor moons and plays catch with them. Each time they fall together under the influence of their mutual gravity, he catches them, absorbs their energy of infall in his springlike muscles, and flings them apart so that they always travel the same distance before returning. It's an enchanting game, but Atlas finds that it's a losing game. When the masses fly back together, they never yield up to him as much energy as he must supply to throw them apart again. Why not?

Gravity waves result from time delay

Say the central point in two words: time delay. Like any force that makes itself felt through the emptiness of space, the force of gravity cannot propagate faster than the speed of light. This limitation imposes a delay on the attraction between the two iron spheres. Alpha, on each little stretch of its outbound path, feels a pull that originated from Beta when the two were a tiny bit *closer* than they are now. The actual force that's slowing Alpha is therefore a tiny bit bigger than we would judge from thinking of them as stationary at their momentary separation. On its return trip inbound along the same little stretch of path, Alpha experiences a helping pull that originated from Beta when the two had a separation slightly *greater* than its present value. The actual force that's speeding Alpha inward is therefore a tiny bit less than we would judge from thinking of them as stationary at their momentary separation. In each stretch of their outbound trip, the two masses have to do more work against the pull of gravity than they get back — in the form of work done on them by gravity — on the same stretch of path inbound. A calculable amount of energy disappears from the local scene on each out-in cycle of Atlas's exercise. Yet the total energy must somehow be conserved. Therefore the very gravity that steals energy from Atlas and his iron, or from any two

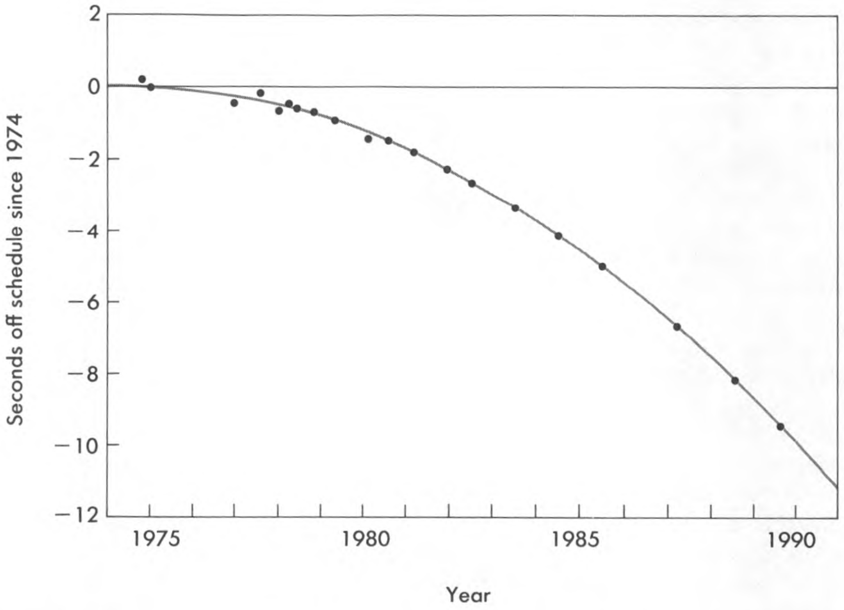


FIGURE 9-7. *Two whirling neutron stars furnish a giant clock, whose time-keeping hand is the line, ever-turning, that separates the centers of those two stars. That hand does not today keep the "slow" schedule (straight horizontal line) one might have expected from its timing as measured in 1974. The downward sloping curve shows gravity-wave theory's prediction of the shortening in the time required to accumulate any specified number of revolutions. The dots show the actual observed shortening in that time.*

masses that rapidly change their relative position, must somehow all the time be transporting the stolen energy to the far-away. That inescapable theft of energy is in its quality, its directional distribution, and its magnitude none other than what Einstein had treated long before under the head of *gravity radiation* and what we now call gravity waves.

Atlas couldn't "see" those gravity waves. Neither have we today yet succeeded in detecting directly the gravity waves we feel sure must be radiating from sources dotted here and there in the galaxy and in the universe. However, we have an exciting indirect confirmation that gravity waves exist—not through their action on any receptor, but through the energy they carry away from a whirling pair of neutron stars. That particular "binary pulsar" first revealed itself to Joseph H. Taylor, Jr., and Russell A. Hulse by periodic pulses of radio waves picked up on the huge disklike antenna at Arecibo in Puerto Rico. As one of these neutron stars spins on its axis, its magnetic field spins with it, giving timing comparable in accuracy to the best atomic clock ever built (Box 9-2). Thanks to this happy circumstance, Taylor and his colleagues have been able to follow the ever-shortening separation of the two stars and the ever-higher speed they attain as they slowly spiral in toward an ultimate catastrophe some 400 million years from now. The timing of the orbits gives us a measure of energy lost as the stars spiral in. No reasonable way has ever been found to account for the thus observed loss of energy except gravitational radiation. As of September 1989, 14 years after first observation, this loss of energy agrees with the rate predicted by theory to better than one percent (Figure 9-7).

Gravity waves steal energy from orbiting neutron stars

Gravity waves and pulses of gravity radiation are sweeping over us all the time from sources of many kinds out in space. Detecting them, however, we are no better than the primitive jungle dweller unable to detect and even totally unaware of the radio waves that carry past her every minute of the day music, words, and messages. However, experimentalists are working out ingenious technology and building detector instrumentation of ever-growing sensitivity (Figures 9-8 and 9-9). Few among them have any doubt of their ability to detect pulses of gravity radiation from one or another star catastrophe by sometime in the first decade of the twenty-first century.

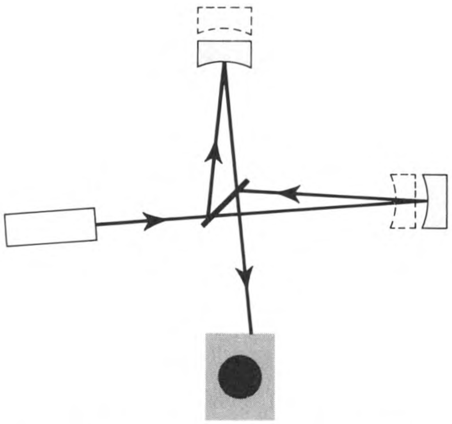


FIGURE 9-8. *The proposed MIT–Caltech gravity-wave detector will (1) use the beam from a laser (left), (2) split it by a device (center) analogous to a half-silvered mirror, (3) send one half-strength beam to one faraway mirror (top) and the other to the other faraway mirror (right), (5) allow these beams to undergo many many reflections (not shown), and (6) recombine them at the detector (bottom). A gravity-wave incident on Earth will slightly shorten the 4-kilometer distance to the one mirror and slightly lengthen the 4-kilometer distance to the other mirror. This relative alteration in the path length of the laser beams, if big enough, amplified enough, and picked up by detectors sensitive enough, will reveal the passage of the gravity wave.*

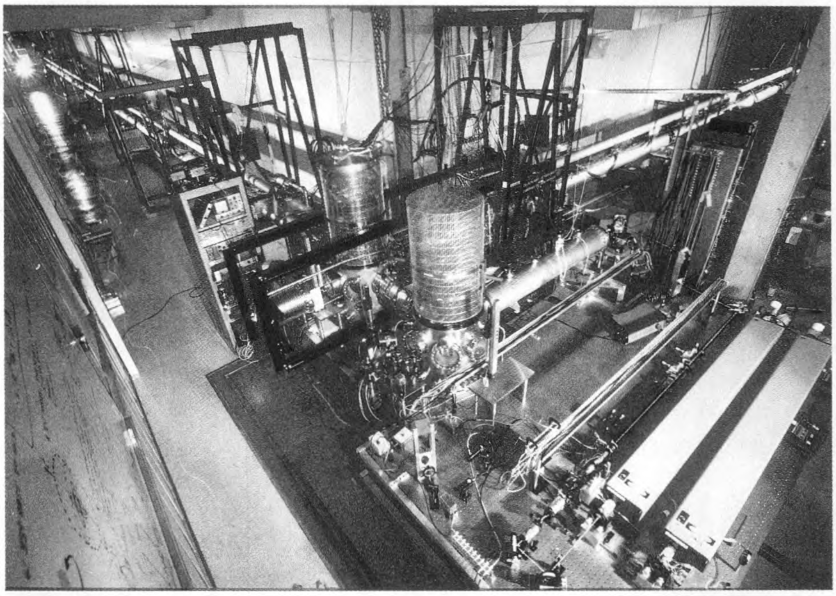


FIGURE 9-9. ***Prototype gravity-wave detector, California Institute of Technology, Pasadena.*** *The laser beam is tailored (lower right) for entry into the beam splitter (located where the two long light pipes meet, just to the left of center in the photograph). The mirrors at the ends of these two evacuated light pipes lie outside the boundary of the photograph.*

Astronomy uses signals of many kinds — light, radio waves, and X-rays among them — to reveal the secrets of the stars. Of all signals from a star, none comes out from deeper in the interior than a gravity wave. Among all violent events to be probed deeply by a gravity wave, none is more fascinating than the dance of death of two compact stars as they whirl around each other and undergo total collapse into . . . a black hole!

9.8 BLACK HOLE

over the edge with a scream of radiation

A black hole is a domain whose mass is so tightly compacted that nothing can escape from it, not even light. Everything that falls in is caught without hope of escape (Figure 9-10).

"Escape velocity c" implies black hole

To fire a missile from Moon's surface so that it escapes that satellite's attraction demands a speed of 2.38 kilometers per second or greater. The critical speed for escape from Earth — in the absence of drag from the atmosphere — is 11.2 kilometers per second. When the object does not rotate and is so compact that even light cannot escape, the "effective radius" or so-called "horizon radius" is

$$\begin{aligned}(\text{effective radius}) &= \frac{\begin{pmatrix}\text{circumference of region}\\ \text{out of which}\\ \text{light cannot escape}\end{pmatrix}}{2\pi}\\ &= 2 \times (1.47\ \text{kilometers}) \times \begin{pmatrix}\text{mass of black hole}\\ \text{expressed in}\\ \text{number of Sun masses}\end{pmatrix}\end{aligned}$$

Black hole still exerts "pull" of gravity

When a star or cloud of matter collapses to a black hole it disappears from view as totally as the Cheshire cat did in Alice in Wonderland. The cat, however, left its grin behind; and the black hole — via the effect of spacetime curvature that we call gravity — exerts as much "pull" as ever on normal stars in orbit around it. They are like participants in a formal dance with lights turned low. Only the white dress of the girl is visible as she whirls around in the arms of her black-suited companion. From the

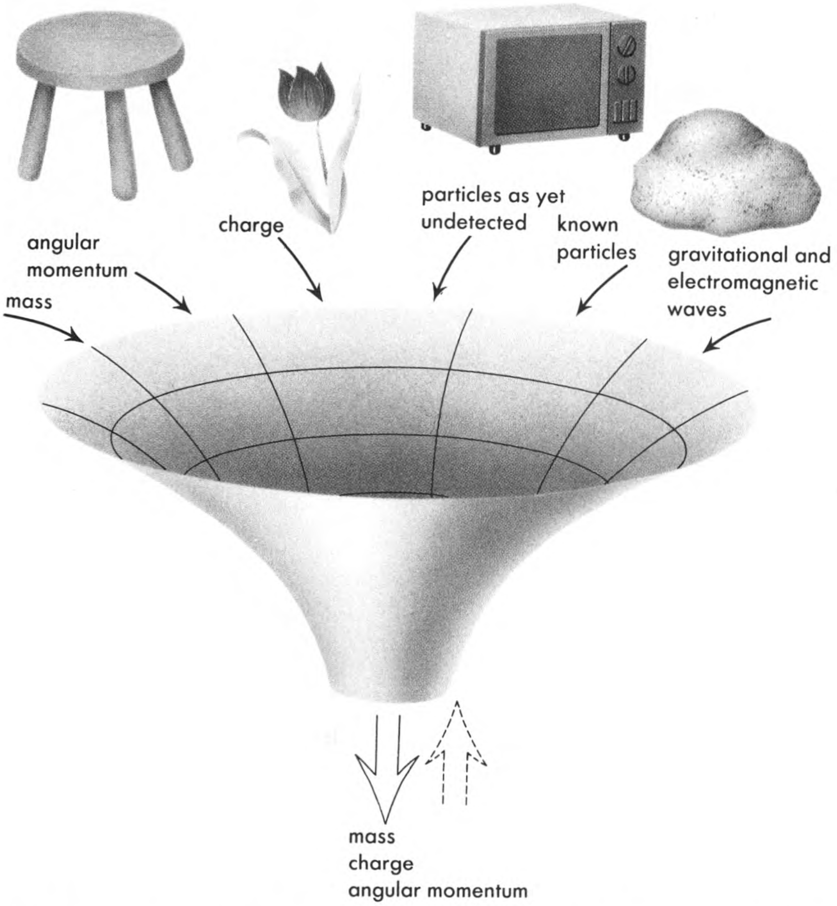


FIGURE 9-10. *Whatever objects fall into a black hole, they possess at the end — as seen from outside — only mass, angular momentum, and electric charge. Not one other characteristic of any in-falling object remains to betray its past — not a hair. This leads to the saying, "A black hole has no hair."*

speed of the girl and the size of the circle in which she swirls, we know something of the mass of the invisible companion. By such reasoning it was possible to conclude by 1972 that the optically invisible companion of one long-known star has a mass of the order of 9.5 solar masses.

Cygnus X-1: A black hole?

This remarkable object came first to attention because in December 1971 the Uhuru orbiting X-ray observatory detected X-ray pulsations with time scales from one tenth to tens of seconds from an object located in the Cygnus region close to the known star. Why does it give off X-rays? And why does the intensity of the X-rays vary rapidly from instant to instant? The gas wind from the visible companion varies from instant to instant like the smoke from a factory chimney. This gas, falling on a compact object, gets squeezed. To picture the how and why of this squeeze, look from a low-flying plane at the streams of automobiles converging from many directions on a football stadium for a Saturday afternoon game. The particles and the gas are pushed together as surely as the cars in the traffic. The compression of the traffic raises the temper of the driver, and the compression of the gas raises its temperature as air is heated when pumped in a bicycle pump. However, because the gas falls from an object of millions of kilometers in size to one a few kilometers across, the compression is so stupendous that the temperature rises far above any normal star temperature, and X-rays come off.

The time scale of the fluctuations in X-ray intensity depends on the size of the object that is picking up the star smoke, a size less by a fantastic factor than that of any normal star. Could the object be a white dwarf (Box 9-2)? No, because such a star would be

TABLE 9-1

BLACK HOLES FOR WHICH THERE WAS SUBSTANTIAL EVIDENCE AS OF SEPTEMBER 1989

(Uncertainties in masses are of the order of 20 to 50 percent.)

Astronomical designation of black hole	Mass (in solar masses)
Cygnus X-1	9.5
LMC X-1	2.6
AO 620-00	3.2
LMC X-3	7.0
SS 433	4.3
Black hole at center of our galaxy	3.5×10^6

visible. A neutron star? No, because even matter compressed so tightly that it is transformed to neutrons cannot support itself against gravity if it has a mass much over two solar masses. No escape has been found from concluding that Cygnus X-1 is a black hole. This great discovery transformed black holes from pencil-and-paper objects into a lively and ever-growing part of modern astrophysics (Table 9-1).

Black hole at center of our galaxy?

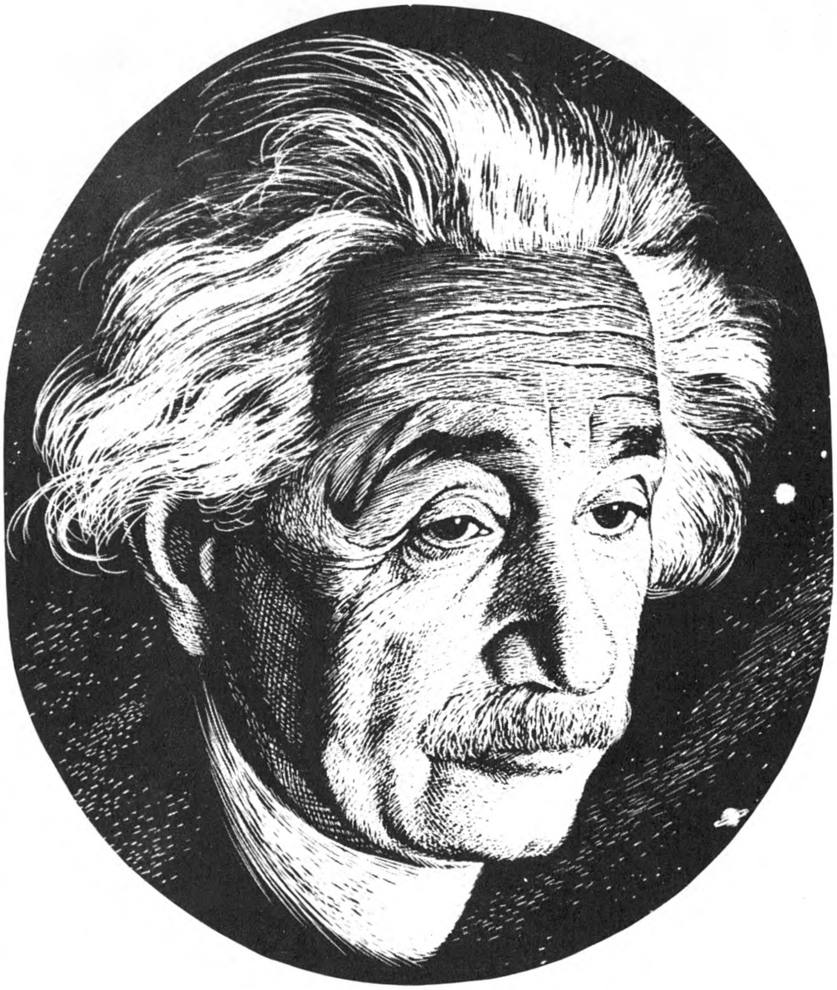
Much attention went in the 1980s to a presumptive black hole with a mass of about three and a half million times the solar mass and a horizon radius of about ten million kilometers. It floats at the center of our galaxy, the Milky Way. Around it buzz visible stars of the everyday kind, most of them fated to fall eventually into that black hole and increase its mass and size. That stars close to the center of our galaxy go around as fast as they do is one of the best indicators we have for the presence, and one of the best measures we have for the mass, of the central black hole, which is itself invisible.

Quasar energy output from matter swirling into black hole?

In contrast to dead solitary black holes, the most powerful source of energy we know or conceive or see in all the universe is a black hole of many millions of solar masses, gulping down enormous amounts of matter swirling around it. Maarten Schmidt, working at the Mount Palomar Observatory in 1956, was the first to uncover evidence for these quasistellar objects, or quasars, starlike sources of light located not billions of kilometers but billions of *light-years* away. Despite being far smaller than any galaxy, the typical quasar manages to put out more than a hundred times as much energy as our own Milky Way, with its hundred billion stars. Quasars, unsurpassed in brilliance and remoteness, we call lighthouses of the heavens.

High-efficiency conversion of gravitational energy to radiation

Observation and theory have come together to explain in broad outline how a quasar operates. A black hole of some hundreds of millions of solar masses, itself built by accretion, accretes more mass from its surroundings. The incoming gas, and stars-converted-to-gas, does not fall in directly, any more than the water rushes directly down the bathtub drain when the plug is pulled. Which way the gas swirls is a matter of chance or past history or both, but it does swirl. This gas, as it goes round and round, slowly makes its way inward to regions of ever-stronger gravity. Thus compressed, and by this compression heated, the gas breaks up into electrons — that is negative ions — and positive ions, linked by magnetic fields of force into a gigantic accretion disk. Matter little by little makes its way to the inner boundary of this accretion disk and then, in a great swoop, falls into the black hole, on its way crossing the horizon, the surface of no return. During that last swoop, hold on the particle is relinquished. Therefore, the chance is lost to extract as energy the full 100 percent of the mass of each infalling bit of matter. However, magnetic fields do hold onto the ions effectively enough for long enough to extract, as energy, several percent of the



Hal McIntosh. Courtesy of *The Saturday Review.*

ALBERT EINSTEIN

Ulm, Germany, March 14, 1879—Princeton, New Jersey, April 18, 1955

"Newton himself was better aware of the weaknesses inherent in his intellectual edifice than the generations which followed him. This fact has always roused my admiration."

★ ★ ★

"Only the genius of Riemann, solitary and uncomprehended, had already won its way by the middle of the last century to a new conception of space, in which space was deprived of its rigidity, and in which its power to take part in physical events was recognized as possible."

★ ★ ★

"All of these endeavors are based on the belief that existence should have a completely harmonious structure. Today we have less ground than ever before for allowing ourselves to be forced away from this wonderful belief."

mass. In contrast, neither nuclear fission nor nuclear fusion is able to obtain a conversion efficiency of more than a fraction of a percent. Of all methods to convert bulk matter into energy, no one has ever seen evidence for a more effective process than accretion into a black hole, and no one has even been able to come up with a more feasible scheme for one.

Of all the features of black hole physics in action, none is more spectacular than a quasar. And no lighthouse of the skies gives more dramatic evidence of the scale of the universe.

9.9 THE COSMOS

a final crunch?

Expanding universe: Evidence for big bang beginning

The more distant quasars and galaxies are, the greater the speed with which they are observed to be receding from us. This expansion argues that somewhere between ten and twenty billion years ago the universe began with a big bang, a time before which there was no time.

We see around us relics of the big bang, not only today's rapidly receding galaxies but also today's abundance of the chemical elements—some among them still radioactive, the "still warm ashes of creation" (V. F. Weisskopf)—and today's greatly cooled but still all-pervasive "primordial cosmic fireball radiation." We now believe that in the first instants of its life, the entire universe filled an infinitesimally small space of enormous density and temperature where matter and energy fused in a homogeneous soup. Immediately the universe began expanding. After about 10^{-6} seconds it had cooled enough that subatomic particles condensed from the matter–energy soup. In the first three minutes after the big bang, neutrons and protons combined to make heavier elements. Eons later stars and galaxies formed. Never since has the universe paused in its continual spread outward.

"Open" universe expanding forever?
Or "closed" universe that recontracts to crunch?
An open question!

Will the universe continue expanding forever? Or will its expansion slow, halt, and turn to contraction and crunch (Table 9-2), a crunch similar in character but on a far larger scale than what happens in the formation of a black hole? Great question! No one who cares deeply about this question can fail to celebrate each week that week's astrophysical advances: instruments, observations, conclusions.

We have come to the end of our journey. We have seen gravity turned to float, space and time meld into spacetime, and spacetime transformed from stage to actor. We have examined how spacetime grips mass, telling it how to move, and how mass grips spacetime, telling it how to curve. Of all the indications that existence at bottom has a simplicity beyond anything we imagine today, there is none more inspiring than the unsurpassed simplicity of gravity as we now see it.

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Extended portions of this chapter were copied (and sometimes modified) from John Archibald Wheeler, *Journey Into Gravity and Spacetime* (Scientific American Library, a division of HPHLP, New York, 1990).

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How Newton came only in stages to the solution of the problem of fall is told nowhere with such care for the fascinating documentation as in Alexander Koyre, "A Documentary History of the Problem of Fall from Kepler to Newton," *Transactions of the American Philosophical Society,* Volume 45, Part 4 (1955).

Keynes quotation under Newton portrait: Reprinted by permission of the publisher, Horizon Press, from *Essays in Biography* by John Maynard Keynes, copyright 1951.

TABLE 9-2

A CLOSED-MODEL UNIVERSE COMPATIBLE WITH OBSERVATION

Radius at phase of maximum expansion	18.9×10^9 light-years or 1.79×10^{26} meters
Time from start to maximum size	29.8×10^9 years or 2.82×10^{26} meters
Radius today	13.2×10^9 light-years
Time from start to today's size	10.0×10^9 years
Time it would have taken from start to today's size if the entire expansion had occurred at today's slowed rate of expansion	20.0×10^9 years
Present expansion rate	An extra increment of recession velocity of 15.0 kilometers/second for every extra million light-years of remoteness of the galactic cluster
Fraction of the way around the 3-sphere universe from which we can in principle receive light today	$\frac{113.2 \text{ degrees}}{180 \text{ degrees}} = 62.9\%$
Fraction of the matter in the 3-sphere universe that has been able to communicate with us so far	74.4%
Number of new galaxies that come into view on average every three days	One!
Average mass density today	14.8×10^{-27} kilogram/meter^3
Average mass density at phase of maximum expansion	5.0×10^{-27} kilogram/meter^3
Rate of increase of volume today	1.82×10^{62} meters^3/second
Amount of mass	$M_{\text{conv}} = 5.68 \times 10^{53}$ kilograms In geometric units: $M = GM_{\text{conv}}/c^2 = 4.21 \times 10^{26}$ meters
Equivalent number of suns like ours	2.86×10^{23}
Equivalent number of galaxies like ours	1.6×10^{12}
Equivalent number of baryons (neutrons and protons)	3.39×10^{80}
Total time, big bang to big crunch	59.52×10^9 years

Figure 9-2: Figure and data from *Journal of Spacecraft,* Volume 11 (September 1974), pages 637 – 644, published by the American Institute of Aeronautics and Astronautics. Data also from D. B. De Bra, *APL Technical Digest,* Volume 12: pages 14 – 26.

Figure 9-3 from *Philosophiae Naturalis Principia Mathematica* (Joseph Streater, London, July 5, 1686); Motte translation into English revised and edited by Florian Cajori and published in two paperback volumes (University of California Press, Berkeley, 1962). This is also the source of the quote in Section 9.6: "Absolute space, in its own nature, without relation to anything external, remains always similar and immobile."

Three quotations under the Einstein picture come from Albert Einstein, *Essays in Science* (Philosophical Library, New York, 1934).

Quotation in Section 9.6: "Rather than have one global frame with gravitational forces we have many local frames without gravitational forces." Steven Schutz, in January 1966 final examination in course in relativity, Princeton University.

For an exciting and readable overview of the experimental proofs of Einstein's general relativity theory, see Clifford M. Will, *Was Einstein Right? Putting General Relativity to the Test* (Basic Books, New York, 1986). In particular (Chapter 10, pages 181 – 206), he describes at some length the emission of gravity waves by the binary pulsar system studied by Joseph H. Taylor, Jr., and Russell A. Hulse.