CHAPTER 1

SPACETIME: OVERVIEW

Our imagination is stretched to the utmost, not, as in fiction, to imagine things which are not really there, but just to comprehend those things which are there.

Richard P. Feynman

1.1 PARABLE OF THE SURVEYORS

disagree on northward and eastward separations; agree on *distance*

Daytime surveyor uses magnetic north

Once upon a time there was a Daytime surveyor who measured off the king's lands. He took his directions of north and east from a magnetic compass needle. Eastward separations from the center of the town square he measured in meters. The northward direction was sacred. He measured northward separations from the town square in a different unit, in miles. His records were complete and accurate and were often consulted by other Daytimers.

Nighttime surveyor uses North-Star north

A second group, the Nighttimers, used the services of another surveyor. Her north and east directions were based on a different standard of north: the direction of the North Star. She too measured separations eastward from the center of the town square in meters and sacred separations northward in miles. The records of the Nighttime surveyor were complete and accurate. Marked by a steel stake, every corner of a plot appeared in her book, along with its eastward and northward separations from the town square.

Daytimers and Nighttimers did not mix but lived mostly in peace with one another. However, the two groups often disputed the location of property boundaries. Why? Because a given corner of the typical plot of land showed up with different numbers in the two record books for its eastward separation from the town center, measured in meters (Figure 1-1). Northward measurements in miles also did not agree between the two record books. The differences were small, but the most careful surveying did not succeed in eliminating them. No one knew what to do about this single source of friction between Daytimers and Nighttimers.

One fall a student of surveying turned up with novel open-mindedness. Unlike all previous students at the rival schools, he attended both. At Day School he learned

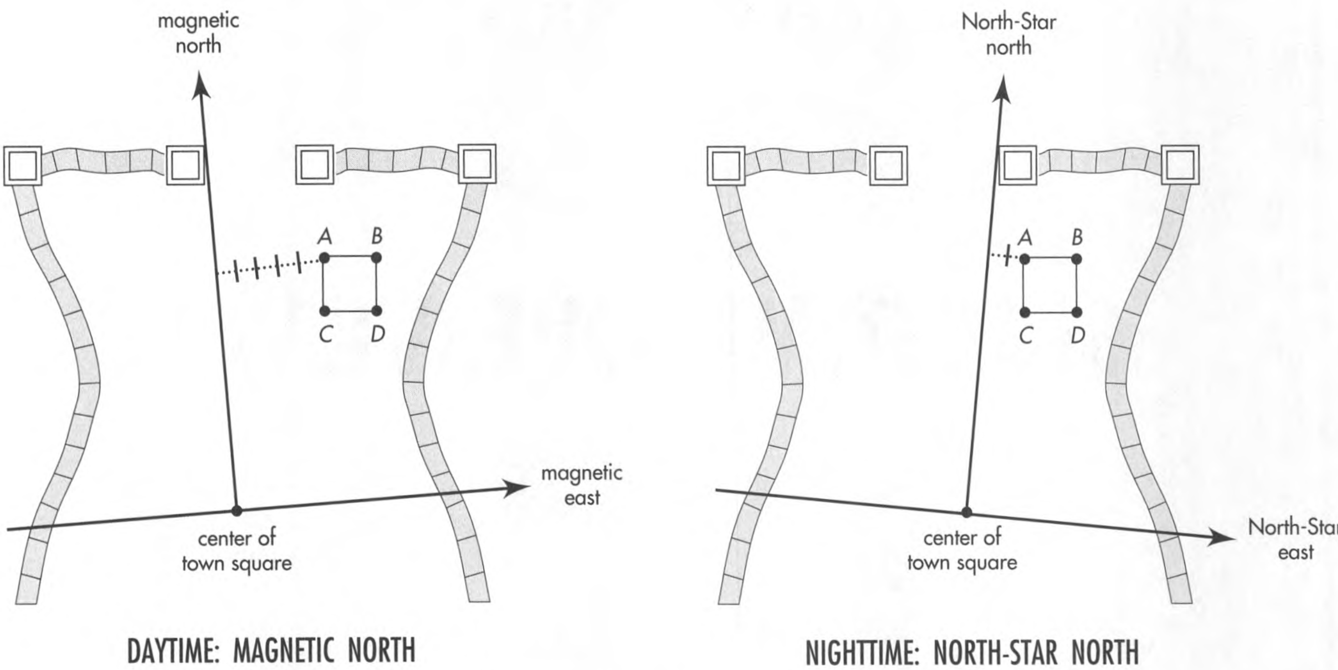


FIGURE 1-1. ***The town as plotted by Daytime and Nighttime surveyors.*** *Notice that the line of Daytime magnetic north just grazes the* left *side of the north gate, while the line of Nighttime North-Star north just grazes the* right *side of the same gate. Steel stakes* A, B, C, D *driven into the ground mark the corners of a disputed plot of land. As shown, the eastward separation of stake* A *from the north–south line measured by the Daytime surveyor is different from that measured by the Nighttime surveyor.*

from one expert his method of recording locations of gates of the town and corners of plots of land based on magnetic north. At Night School he learned the other method, based on North-Star north.

As days and nights passed, the student puzzled more and more in an attempt to find some harmonious relationship between rival ways of recording location. His attention was attracted to a particular plot of land, the subject of dispute between Daytimers and Nighttimers, and to the steel stakes driven into the ground to mark corners of this disputed plot. He carefully compared records of the two surveyors (Figure 1-1, Table 1-1).

Student converts miles to meters

In defiance of tradition, the student took the daring and heretical step of converting northward measurements, previously expressed always in miles, into meters by multiplying with a constant conversion factor k. He found the value of this conversion factor to be $k = 1609.344$ meters/mile. So, for example, a northward separation of 3 miles could be converted to $k \times 3$ miles $= 1609.344$ meters/mile $\times$ 3 miles $= 4828.032$ meters. "At last we are treating both directions the same!" he exclaimed.

Next the student compared Daytime and Nighttime measurements by trying various combinations of eastward and northward separation between a given stake and the center of the town square. Somewhere the student heard of the Pythagorean Theorem, that the sum of squares of the lengths of two perpendicular legs of a right triangle equals the square of the length of the hypotenuse. Applying this theorem, he discovered that the expression

$$\begin{array}{ccc} & \textbf{Daytime} & & \textbf{Daytime} \\ & \left[k \times \begin{pmatrix}\text{northward}\\ \text{separation}\\ \text{(miles)}\end{pmatrix}\right]^2 & + & \begin{bmatrix}\text{eastward}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2 \end{array} \tag{1-1}$$

TABLE 1-1

TWO DIFFERENT SETS OF RECORDS; SAME PLOT OF LAND

	Daytime surveyor's axes oriented to magnetic north		*Nighttime surveyor's axes oriented to North-Star north*	
	Eastward (meters)	Northward (miles)	Eastward (meters)	Northward (miles)
Town square	0	0	0	0
Corner stakes:				
Stake A	4010.1	1.8330	3950.0	1.8827
Stake B	5010.0	1.8268	4950.0	1.8890
Stake C	4000.0	1.2117	3960.0	1.2614
Stake D	5000.0	1.2054	4960.0	1.2676

based on Daytime measurements of the position of steel stake C had exactly the same numerical value as the quantity

$$\overset{\textbf{Nighttime}}{\left[k \times \begin{pmatrix}\text{northward}\\ \text{separation}\\ \text{(miles)}\end{pmatrix}\right]^2} + \overset{\textbf{Nighttime}}{\begin{bmatrix}\text{eastward}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2} \tag{1-2}$$

computed from the readings of the Nighttime surveyor for stake C (Table 1-2). He

TABLE 1-2

"INVARIANT DISTANCE" FROM CENTER OF TOWN SQUARE TO STAKE *C*

(Data from Table 1-1)

Daytime measurements		Nighttime measurements	
Northward separation 1.2117 miles		Northward separation 1.2614 miles	
Multiply by $k = 1609.344$ meters/mile to convert to meters: 1950.0 meters		Multiply by $k = 1609.344$ meters/mile to convert to meters: 2030.0 meters	
Square the value	3,802,500 (meters)^2	Square the value	4,120,900 (meters)^2
Eastward separation 4000.0 meters		Eastward separation 3960.0 meters	
Square the value and add	+ 16,000,000 (meters)^2	Square the value and add	+ 15,681,600 (meters)^2
Sum of squares	= 19,802,500 (meters)^2	Sum of squares	= 19,802,500 (meters)^2
Expressed as a number squared	= (4450 meters)^2	Expressed as a number squared	= (4450 meters)^2
This is the square of what measurement?	**4450 meters**	This is the square of what measurement?	**4450 meters**

SAME
DISTANCE
from center of Town Square

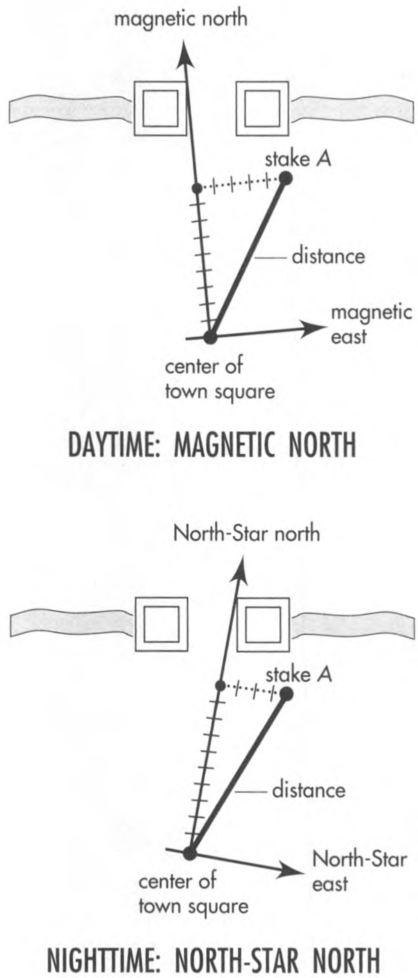


FIGURE 1-2. *The distance between stake* A *and the center of the town square has the same value for Daytime and Nighttime surveyors, even though the northward and eastward separations, respectively, are not the same for the two surveyors.*

tried the same comparison on recorded positions of stakes A, B, and D and found agreement here too. The student's excitement grew as he checked his scheme of comparison for all stakes at the corners of disputed plots—and found everywhere agreement.

Discovery: Invariance of distance

Flushed with success, the student methodically converted all northward measurements to units of meters. Then the student realized that the quantity he had calculated, the numerical value of the above expressions, was not only the same for Daytime and Nighttime measurements. It was also the square of a length: (meters)^2. He decided to give this length a name. He called it the **distance** from the center of town.

$$(\text{distance})^2 = \begin{bmatrix} \text{northward} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}^2 + \begin{bmatrix} \text{eastward} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}^2 \qquad \text{(1-3)}$$

He said he had discovered the **principle of invariance of distance;** he reckoned exactly the same value for distance from Daytime measurements as from Nighttime measurements, despite the fact that the two sets of surveyors' numbers differed significantly (Figure 1-2).

After some initial confusion and resistance, Daytimers and Nighttimers welcomed the student's new idea. The invariance of distance, along with further results, made it possible to harmonize Daytime and Nighttime surveys, so everyone could agree on the location of each plot of land. In this way the last source of friction between Daytimers and Nighttimers was removed.

1.2 SURVEYING SPACETIME

disagree on separations in space and time; agree on spacetime *interval*

The Parable of the Surveyors illustrates the naive state of physics before the discovery of **special relativity** by Einstein of Bern, Lorentz of Leiden, and Poincaré of Paris. Naive in what way? Three central points compare physics at the turn of the twentieth century with surveying before the student arrived to help Daytimers and Nighttimers.

The second: A sacred unit

First, surveyors in the mythical kingdom measured northward separations in a sacred unit, the mile, different from the unit used in measuring eastward separations. Similarly, people studying physics measured time in a sacred unit, called the second, different from the unit used to measure space. No one suspected the powerful results of using the same unit for both, or of squaring and combining space and time separations when both were measured in meters. Time in meters is just the time it takes a light flash to go that number of meters. The conversion factor between seconds and meters is the speed of light, $c = 299{,}792{,}458$ meters/second. The velocity of light c (in meters/second) multiplied by time t (in seconds) yields ct (in meters).

Speed of light converts seconds to meters

The speed of light is the only natural constant that has the necessary units to convert a time to a length. Historically the value of the speed of light was regarded as a sacred number. It was not recognized as a mere conversion factor, like the factor of conversion between miles and meters — a factor that arose out of historical accident in humankind's choice of units for space and time, with no deeper physical significance.

Second, in the parable northward readings as recorded by two surveyors did not differ much because the two directions of north were inclined to one another by only the small angle of 1.15 degrees. At first our mythical student thought that small differences between Daytime and Nighttime northward measurements were due to surveying error alone. Analogously, we used to think of the separation in time between two electric sparks as the same, regardless of the motion of the observer. Only with the publication of Einstein's relativity paper in 1905 did we learn that the separation in time between two sparks really has different values for observers in different states of motion — in different **frames.**

Time between events: Different for different frames

Think of John standing quietly in the front doorway of his laboratory building. Suddenly a rocket carrying Mary flashes through the front door past John, zooms down the middle of the long corridor, and shoots out the back door. An antenna projects from the side of Mary's rocket. As the rocket passes John, a spark jumps across the 1-millimeter gap between the antenna and a pen in John's shirt pocket. The rocket continues down the corridor. A second spark jumps 1 millimeter between the antenna and the fire extinguisher mounted on the wall 2 meters farther down the corridor. Still later other metal objects nearer the rear receive additional sparks from the passing rocket before it finally exits through the rear door.

One observer uses laboratory frame

John and Mary each measure the lapse of time between "pen spark" and "fire-extinguisher spark." They use accurate and fast electronic clocks. John measures this time lapse as 33.6900 thousand-millionths of a second (0.0000000336900 second $= 33.6900 \times 10^{-9}$ second). This equals 33.6900 **nanoseconds** in the terminology of high-speed electronic circuitry. (One nanosecond $= 10^{-9}$ second.) Mary measures a slightly different value for the time lapse between the two sparks, 33.0228 nanoseconds. For John the fire-extinguisher spark is separated in space by 2.0000 meters from the pen spark. For Mary in the rocket the pen spark and fire-extinguisher spark occur at the same place, namely at the end of her antenna. Thus for her their space separation equals zero.

Another observer uses rocket frame

Later, laboratory and rocket observers compare their space and time measurements between the various sparks (Table 1-3). Space locations and time lapses in both frames are measured from the pen spark.

TABLE 1-3

SPACE AND TIME LOCATIONS OF THE SAME SPARKS AS SEEN BY TWO OBSERVERS

	Distance and time between sparks as measured by observer who is standing in laboratory (John)		*moving by in rocket (Mary)*	
	Distance (meters)	Time (nanoseconds)	Distance (meters)	Time (nanoseconds)
Reference spark (pen spark)	0	0	0	0
Spark *A* (fire-extinguisher spark)	2.0000	33.6900	0	33.0228
Spark *B*	3.0000	50.5350	0	49.5343
Spark *C*	5.0000	84.2250	0	82.5572
Spark *D*	8.0000	134.7600	0	132.0915

Discovery: Invariance of spacetime interval

The third point of comparison between the Parable of the Surveyors and the state of physics before special relativity is this: The mythical student's discovery of the concept of distance is matched by the Einstein – Poincaré discovery in 1905 of the **invariant spacetime interval** (formal name **Lorentz interval,** but we often say just **interval**), a central theme of this book. Let each time measurement in seconds be converted to meters by multiplying it by the "conversion factor c," the speed of light:

$$\begin{aligned} c &= 299{,}792{,}458 \text{ meters/second} = 2.99792458 \times 10^8 \text{ meters/second} \\ &= 0.299792458 \times 10^9 \text{ meters/second} = 0.299792458 \text{ meters/nanosecond} \end{aligned}$$

Then the square of the spacetime interval is calculated from the laboratory observer's measurements by *subtracting* the square of the space separation from the square of the time separation. Note the minus sign in equation (1-4).

$$(\text{interval})^2 = \overbrace{\left[c \times \begin{pmatrix}\text{time}\\ \text{separation}\\ (\text{seconds})\end{pmatrix}\right]^2}^{\textbf{Laboratory}} - \overbrace{\begin{bmatrix}\text{space}\\ \text{separation}\\ (\text{meters})\end{bmatrix}^2}^{\textbf{Laboratory}} \qquad (1\text{-}4)$$

The rocket calculation gives exactly the same value of the interval as the laboratory calculation,

$$(\text{interval})^2 = \overbrace{\left[c \times \begin{pmatrix}\text{time}\\ \text{separation}\\ (\text{seconds})\end{pmatrix}\right]^2}^{\textbf{Rocket}} - \overbrace{\begin{bmatrix}\text{space}\\ \text{separation}\\ (\text{meters})\end{bmatrix}^2}^{\textbf{Rocket}} \qquad (1\text{-}5)$$

even though the respective space and time separations are not the same. Two observers find different space and time separations, respectively, between pen spark and fire-extinguisher spark, but when they calculate the spacetime interval between these sparks their results agree (Table 1-4).

The student surveyor found that invariance of distance was most simply written with both northward and eastward separations expressed in the same unit, the meter. Likewise, invariance of the spacetime interval is most simply written with space and

TABLE 1-4

"INVARIANT SPACETIME INTERVAL" FROM REFERENCE SPARK TO SPARK A

(Data from Table 1-3)

Laboratory measurements		Rocket measurements	
Time lapse 33.6900×10^{-9} seconds = 33.6900 nanoseconds		Time lapse 33.0228×10^{-9} seconds = 33.0228 nanoseconds	
Multiply by $c = 0.299792458$ meters per nanosecond to convert to meters: 10.1000 meters		Multiply by $c = 0.299792458$ meters per nanosecond to convert to meters: 9.9000 meters	
Square the value	102.010 (meters)^2	Square the value	98.010 (meters)^2
Spatial separation 2.000 meters		Spatial separation zero	
Square the value and subtract	− 4.000 (meters)^2	Square the value and subtract	− 0
Result of subtaction	= 98.010 (meters)^2	Result of subtaction	= 98.010 (meters)^2
expressed as a number squared	= (9.900 meters)^2	expressed as a number squared	= (9.900 meters)^2
This is the square of what measurement?	**9.900 meters**	This is the square of what measurement?	**9.900 meters**

SAME SPACETIME
INTERVAL
from the reference event

time separations expressed in the same unit. Time is converted to meters: t (meters) = $c \times t$ (seconds). Then the interval appears in simplified form:

$$(\text{interval})^2 = \begin{bmatrix} \text{time} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}^2 - \begin{bmatrix} \text{space} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}^2 \tag{1-6}$$

Space and time are part of spacetime

The **invariance of the spacetime interval**—its independence of the state of motion of the observer—forces us to recognize that time cannot be separated from space. Space and time are part of a single entity, **spacetime.** Space has three dimensions: northward, eastward, and upward. Time has one dimension: onward! The interval combines all four dimensions in a single expression. The geometry of spacetime is truly four-dimensional.

To recognize the unity of spacetime we follow the procedure that makes a landscape take on depth—we look at it from several angles. That is why we compare space and time separations between events A and B as recorded by two different observers in relative motion.

Why the minus sign in the equation for the interval? Pythagoras tells us to ADD the squares of northward and eastward separations to get the square of the distance. Who tells us to SUBTRACT the square of the space separation between events from the square of their time separation in order to get the square of the spacetime interval?

Shocked? Then you're well on the way to understanding the new world of very fast motion! This world goes beyond the three-dimensional textbook geometry of Euclid, in which distance is reckoned from a sum of squares. In this book we use another kind of geometry, called **Lorentz geometry,** more real, more powerful than Euclid for the world of the very fast. In Lorentz geometry the squared space separation is combined with the squared time separation in a new way—by *subtraction*. The result is the square of a new unity called the *spacetime interval* between events. The numerical value of this interval is *invariant,* the same for all observers, no matter how fast they are moving past one another. Proof? Every minute of every day an experiment somewhere in the world demonstrates it. In Chapter 3 we derive the invariance of the spacetime interval—with its minus sign—from experiments. They show the finding that no experiment conducted in a closed room will reveal whether that room is "at rest" or "in motion" (Einstein's Principle of Relativity). We won't wait until then to cash in on the idea of interval. We can begin to enjoy the payoff right now.

SAMPLE PROBLEM 1-1

SPARKING AT A FASTER RATE

Another, even faster rocket follows the first, entering the front door, zipping down the long corridor, and exiting through the back doorway. Each time the rocket clock ticks it emits a spark. As before, the first spark jumps the 1 millimeter from the passing rocket antenna to the pen in the pocket of John, the laboratory observer. The second flash jumps when the rocket antenna reaches a doorknob 4.00000000 meters farther along the hall as measured by the laboratory observer, who records the time between these two sparks as 16.6782048 nanoseconds.

a. What is the time between sparks, measured in meters by John, the laboratory observer?

b. What is the value of the spacetime interval between the two events, calculated from John's laboratory measurements?

c. Predict: What is the value of the interval calculated from measurements in the new rocket frame?

d. What is the distance between sparks as measured in this rocket frame?

e. What is the time (in meters) between sparks as measured in this rocket frame? Compare with the time between the same sparks as measured by John in the laboratory frame.

f. What is the speed of this rocket as measured by John in the laboratory?

SOLUTION

a. Time in meters equals time in nanoseconds multiplied by the conversion factor, the speed of light in meters per nanosecond. For John, the laboratory observer,

$$\begin{aligned}&16.6782048 \text{ nanoseconds} \times 0.299792458 \text{ meters/nanosecond}\\ &\qquad = 5.00000000 \text{ meters}\end{aligned}$$

b. The square of the interval between two flashes is reckoned by subtracting the square of the space separation from the square of the time separation. Using laboratory figures:

$$\begin{aligned}(\text{interval})^2 &= (\text{laboratory time})^2 - (\text{laboratory distance})^2\\ &= (5 \text{ meters})^2 - (4 \text{ meters})^2 = 25 \text{ (meters)}^2 - 16 \text{ (meters)}^2\\ &= 9 \text{ (meters)}^2 = (3 \text{ meters})^2\end{aligned}$$

Therefore the interval between the two sparks has the value 3 meters (to nine significant figures).

c. We strongly assert in this chapter that the **spacetime interval is invariant**— has the same value by whomever calculated. Accordingly, the interval between the two sparks calculated from rocket observations has the same value as the interval (3 meters) calculated from laboratory measurements.

d. From the rocket rider's viewpoint, both sparks jump from the same place, namely the end of her antenna, and so distance between the sparks equals zero for the rocket rider.

e. We know the value of the spacetime interval between two sparks as computed in the rocket frame (**c**). And we know that the interval is computed by subtracting the square of the space separation from the square of the time separation in the rocket frame. Finally we know that the space separation in the rocket frame equals zero (**d**). Therefore the rocket time lapse between the two sparks equals the interval between them:

$$\begin{aligned}(\text{interval})^2 &= (\text{rocket time})^2 - (\text{rocket distance})^2 \\ (3 \text{ meters})^2 &= (\text{rocket time})^2 - (\text{zero})^2\end{aligned}$$

from which 3 meters equals the rocket time between sparks. Compare this with 5 meters of light-travel time between sparks as measured in the laboratory frame.

f. Measured in the laboratory frame, the rocket moves 4 meters of distance (statement of the problem) in 5 meters of light-travel time (**a**). Therefore its speed in the laboratory is 4/5 light speed. Why? Well, light moves 4 meters of distance in 4 meters of time. The rocket takes longer to cover this distance: 5 meters of time. Suppose that instead of 5 meters of time, the rocket had taken 8 meters of time, twice as long as light, to cover the 4 meters of distance. In that case it would be moving at 4/8—or half—the speed of light. In the present case the rocket travels the 4 meters of distance in 5 meters of time, so it moves at 4/5 light speed. Therefore its speed equals

$$\begin{aligned}&(4/5) \times 2.99792458 \times 10^8 \text{ meters/second} \\ &\qquad = 2.3983397 \times 10^8 \text{ meters/second}\end{aligned}$$

1.3 EVENTS AND INTERVALS ALONE!

tools enough to chart matter and motion without any reference frame

In surveying, the fundamental concept is **place.** The surveyor drives a steel stake to mark the corner of a plot of land—to mark a place. A second stake marks another corner of the same plot—another place. Every surveyor—no matter what his or her standard of north—can agree on the value of the distance between the two stakes, between the two places.

Surveying locates a place

Every stake has its own reality. Likewise the *distance* between every pair of stakes also has its own reality, which we can experience directly by pacing off the straight line from one stake to the other stake. The reading on our pedometer—the distance between stakes—is independent of all surveyors' systems, with their arbitrary choice of north.

More: Suppose we have a table of distances between every pair of stakes. That is all we need! From this table and the laws of Euclidean geometry, we can construct the map of every surveyor (see the exercises for this chapter). Distances between stakes: That is all we need to locate every stake, every place on the map.

Physics locates an event

In physics, the fundamental concept is **event.** The collision between one particle and another is an event, with its own location in spacetime. Another event is the emission of a flash of light from an atom. A third is the impact of the pebble that chips the windshield of a speeding car. A fourth event, likewise fixing in and by itself a location in spacetime, is the strike of a lightning bolt on the rudder of an airplane. An event marks a location in spacetime; it is like a steel stake driven into spacetime.

Every laboratory and rocket observer—no matter what his or her relative velocity—can agree on the spacetime interval between any pair of events.

Wristwatch measures interval directly

Every event has its own reality. Likewise the *interval* between every pair of events also has its own reality, which we can experience directly. We carry our wristwatch at constant velocity from one event to the other one. It is not enough just to pass through the two physical locations—we must pass through the actual *events;* we must be at each event precisely when it occurs. Then the space separation between the two events is zero for us—they both occur at our location. As a result, our wristwatch reads directly the spacetime interval between the pair of events:

$$
\begin{aligned}
(\text{interval})^2 &= \begin{bmatrix}\text{time}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2 - \begin{bmatrix}\text{space}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2 \\
&= \begin{bmatrix}\text{time}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2 - [\text{zero}]^2 = \begin{bmatrix}\text{time}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2 \qquad \text{[wristwatch time]}
\end{aligned}
$$

The time read on a wristwatch carried between two events—the interval between those events—is independent of all laboratory and rocket reference frames.

More: To chart all happenings, we need no more than a table of spacetime intervals between every pair of events. That is all we need! From this table and the laws of Lorentz geometry, it turns out, we can construct the space and time locations of events as observed by every laboratory and rocket observer. Intervals between events: That is all we need to specify the location of every event in spacetime.

"Do science" with intervals alone

In brief, we can completely describe and locate events entirely without a reference frame. We can analyze the physical world—we can "do science"—simply by cataloging every event and listing the interval between it and every other event. The unity of spacetime is reflected in the simplicity of entries in our table: intervals only.

Of course, if we want to use a reference frame, we can do so. We then list in our table the individual northward, eastward, upward, and time separations between pairs of events. However, these laboratory-frame listings for a given pair of events will be different from the corresponding listings that our rocket-frame colleague puts in her table. Nevertheless, we can come to agreement if we use the individual separations to reckon the interval between each pair of events:

$$(\text{interval})^2 = (\text{time separation})^2 - (\text{space separation})^2$$

That returns us to a universal, frame-independent description of the physical world.

When two events both occur at the position of a certain clock, that special clock measures directly the interval between these two events. The interval is called the **proper time** (or sometimes the **local time**). The special clock that records the proper time directly has the name **proper clock** for this pair of events. In this book we often call the proper time the **wristwatch time** and the proper clock the **wristwatch** to emphasize that the proper clock is carried so that it is "present" at each of the two events as the events occur.

In Einstein's German, the word for proper time is *Eigenzeit,* or "own-time," implying "one's very own time." The German word provides a more accurate description than the English. In English, the word "proper" has come to mean "following conventional rules." Proper time certainly does not do that!

Hey! I just thought of something: Suppose two events occur at the same time in my frame but very far apart, for example two handclaps, one in New York City and one in San Francisco. Since they are simultaneous in my frame, the time separation between handclaps is zero. But the space separation is not zero — they are separated by the width of a continent. Therefore the square of the interval is a negative number:

$$\begin{aligned}(\textit{interval})^2 &= (\textit{time separation})^2 - (\textit{space separation})^2 \\ &= (\textit{zero})^2 - (\textit{space separation})^2 = -(\textit{space separation})^2\end{aligned}$$

How can the square of the spacetime interval be negative?

In most of the situations described in the present chapter, there exists a reference frame in which two events occur at the same place. In these cases time separation predominates in all frames, and the interval squared will always be positive. We call these intervals **timelike intervals.**

Euclidean geometry adds squares in reckoning distance. Hence the result of the calculation, distance squared, is always positive, regardless of the relative magnitudes of north and east separations. Lorentz geometry, however, is richer. For your simultaneous handclaps in New York City and San Francisco, space separation between handclaps predominates. In such cases, the interval is called a **spacelike interval** and its form is altered to

$$(\text{interval})^2 = (\text{space separation})^2 - (\text{time separation})^2 \qquad \textbf{[when spacelike]}$$

This way, the squared interval is never negative.

The *timelike* interval is measured directly using a wristwatch carried from one event to the other in a special frame in which they occur at the *same place*. In contrast, a *spacelike* interval is measured directly using a rod laid between the events in a special frame in which they occur at the *same time*. This is the frame you describe in your example.

Spacelike interval or timelike interval: In either case the interval is invariant — has the same value when reckoned using rocket measurements as when reckoned using laboratory measurements. You may want to skim through Chapter 6 where timelike and spacelike intervals are described more fully.

1.4 SAME UNIT FOR SPACE AND TIME: METER, SECOND, MINUTE, OR YEAR

meter for particle accelerators; *minute* for planets; *year* for the cosmos

Measure time in meters

The parable of the surveyors cautions us to use the same unit to measure both space and time. So we use meter for both. Time can be measured in meters. Let a flash of light bounce back and forth between parallel mirrors separated by 0.5 meter of

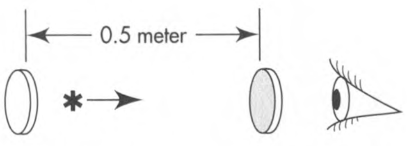


FIGURE 1-3. *This two-mirror "clock" sends to the eye flash after flash, each separated from the next by 1 meter of light-travel time. A light flash (represented by an asterisk) bounces back and forth between parallel mirrors separated from one another by 0.5 meter of distance. The silver coating of the right-hand mirror does not reflect perfectly: It lets 1 percent of the light pass through to the eye each time the light pulse hits it. Hence the eye receives a pulse of light every meter of light-travel time.*

distance (Figure 1-3). Such a device is a "clock" that "ticks" each time the light flash arrives back at a given mirror. Between ticks the light flash has traveled a round-trip distance of 1 meter. Therefore we call the stretch of time between ticks **1 meter of light-travel time** or more simply **1 meter of time.**

One meter of light-travel time is quite small compared to typical time lapses in our everyday experience. Light travels nearly 300 million meters per second (300,000,000 meters/second $= 3 \times 10^8$ meters/second, four fifths of the way to Moon in one second). Therefore one second equals 300 million meters of light-travel time. So 1 meter of light-travel time has the small value of one three-hundred-millionth of a second. [How come? Because (1) light goes 300 million meters in one second, and (2) one three-hundred-millionth of that distance (one meter!) is covered in one three-hundred-millionth of that time.] Nevertheless this unit of time is very useful when dealing with light and with high-speed particles. A proton early in its travel through a particle accelerator may be jogging along at "only" one half the speed of light. Then it travels 0.5 meter of distance in 1 meter of light-travel time.

We, our cars, even our jet planes, creep along at the pace of a snail compared with light. We call a deed quick when we've done it in a second. But a second for light means a distance covered of 300 million meters, seven trips around Earth. As we dance around the room to the fastest music, oh, how slow we look to light! Not zooming. Not dancing. Not creeping. Oozing! That long slow ooze racks up an enormous number of meters of light-travel time. That number is so huge that, by the end of one step of our frantic dance, the light that carries the image of the step's beginning is well on its way to Moon.

Meter officially defined using light speed

In 1983 the General Conference on Weights and Measures officially redefined the meter in terms of the speed of light. **The meter is now defined as the distance that light travels in a vacuum in the fraction 1/299,792,458 of a second.** (For the definition of the second, see Box 3-2.) Since 1983 the speed of light is, *by definition,* equal to $c = 299{,}792{,}458$ meters/second. This makes official the central position of the speed of light as a conversion factor between time and space.

This official action defines distance (meter) in terms of time (second). Every day we use time to measure distance. "My home is only ten minutes (by car) from work." "The business district is a five-minute walk." Each statement implies a speed—the speed of driving or walking—that converts distance to time. But these speeds can vary—for example, when we get caught in traffic or walk on crutches. In contrast, the speed of light in a vacuum does not vary. It always has the same value when measured over time and the same value as measured by every observer.

Measure distance in light-years

We often describe distances to stars and galaxies using a unit of time. These distances we measure in light-years. One light-year equals the distance that light travels in one year. Along with the light-year of space goes the year of time. Here again, space and time are measured in the same units—years. Here again the speed of light is the conversion factor between measures of time and space. From our everyday perspective one light-year of space is quite large, almost 10,000 million million meters: 1 light-year $= 9{,}460{,}000{,}000{,}000{,}000$ meters $= 0.946 \times 10^{16}$ meters. Nevertheless it is a convenient unit for measuring distance between stars. For example, the nearest star to our Sun, Proxima Centauri, lies 4.28 light-years away.

Any common unit of space or time may be used as the same unit for both space *and* time. For example, Table 1-5 gives us another convenient measure of time, seconds, compared with time in meters. We can also measure space in the same units, light-seconds. Our Sun is 499 light-seconds—or, more simply, 499 seconds—of distance from Earth. Seconds are convenient for describing distances and times among events that span the solar system. Alternatively we could use minutes of time and light-minutes of distance: Our Sun is 8.32 light-minutes from Earth. We can also use hours of time and light-hours of distance. In all cases, the speed of light is the conversion factor between units of space and time.

TABLE 1-5

SOME LIGHT-TRAVEL TIMES

	Time in seconds of light-travel time	*Time in meters*
Telephone call one way: New York City to San Francisco via surface microwave link	0.0138	4,139,000
Telephone call one way: New York City to San Francisco via Earth satellite	0.197	59,000,000
Telephone call one way: New York City to San Francisco bounced off Moon	2.51	752,000,000
Flash of light: Emitted by Sun, received on Earth	499.0	149,600,000,000

Expressing time and space in the same unit **meter** is convenient for describing motion of high-speed particles in the confines of the laboratory. Time and space in the same unit **second** (or **minute** or **hour**) is convenient for describing relations among events in our solar system. Time and space in the same unit **year** is convenient for describing relations among stars and among galaxies. In all three arenas spacetime is the stage and special relativity is the spotlight that illuminates the inner workings of Nature.

Use convenient units, the same for space and time

We are not accustomed to measuring time in meters. So as a reminder to ourselves we add a descriptor: meters *of light-travel time*. But the unit of time is still the meter. Similarly, the added words "seconds *of distance*" and "*light*-years" help to remind us that distance is measured in seconds or years, units we usually associate with time. But this unit of distance is really just second or year. The modifying descriptors are for our convenience only. In Nature, space and time form a unity: spacetime!

The words sound OK. The mathematics appears straightforward. The Sample Problems seem logical. But the ideas are so strange! *Why should I believe them? How can invariance of the interval be proved?*

No wonder these ideas seem strange. Particles zooming by at nearly the speed of light—how far this is from our everyday experience! Even the soaring jet plane crawls along at less than one-millionth light speed. Is it so surprising that the world appears different at speeds a million times faster than those at which we ordinarily move with respect to Earth?

The notion of *spacetime interval* distills a wealth of real experience. We begin with interval because it endures: It illuminates observations that range from the core of a nucleus to the center of a black hole. Understand the spacetime interval and you vault, in a single bound, to the heart of spacetime.

Chapter 3 presents a logical proof of the invariance of the interval. Chapter 4 reports a knock-down argument about it. Chapters that follow describe many experiments whose outcomes are totally incomprehensible unless the interval is invariant. Real verification comes daily and hourly in the on-going enterprise of experimental physics.

SAMPLE PROBLEM 1-2
PROTON, ROCK, AND STARSHIP

a. A proton moving at 3/4 light speed (with respect to the laboratory) passes through two detectors 2 meters apart. Events 1 and 2 are the transits through the two detectors. What are the laboratory space and time separations between the two events, in meters? What are the space and time separations between the events in the proton frame?

b. A speeding rock from space streaks through Earth's outer atmosphere, creating a short fiery trail (Event 1) and continues on its way to crash into Sun (Event 2) 10 minutes later as observed in the Earth frame. Take Sun to be 1.4960×10^{11} meters from Earth. In the Earth frame, what are space and time separations between Event 1 and Event 2 in minutes? What are space and time separations between the events in the frame of the rock?

c. In the twenty-third century a starship leaves Earth (Event 1) and travels at 95 percent light speed, later arriving at Proxima Centauri (Event 2), which lies 4.3 light-years from Earth. What are space and time separations between Event 1 and Event 2 as measured in the Earth frame, in years? What are space and time separations between these events in the frame of the starship?

SOLUTION

a. The space separation measured in the laboratory equals 2 meters, as given in the problem. A flash of light would take 2 meters of light-travel time to travel between the two detectors. Something moving at 1/4 light speed would take four times as long: 2 meters/(1/4) = 8 meters of light-travel time to travel from one detector to the other. The proton, moving at 3/4 light speed, takes 2 meters/(3/4) = 8/3 meters = 2.66667 meters of light-travel time between events as measured in the laboratory.

Event 1 and Event 2 both occur at the position of the proton. Therefore the space separation between the two events equals zero in the proton frame. This means that the spacetime interval—the proper time—equals the time between events in the proton frame.

$$\begin{aligned}
(\text{proton time})^2 - (\text{proton distance})^2 &= (\text{interval})^2 = (\text{lab time})^2 - (\text{lab distance})^2 \\
(\text{proton time})^2 - (\text{zero})^2 &= (2.66667 \text{ meters})^2 - (2 \text{ meters})^2 \\
&= (7.1111 - 4)\ (\text{meters})^2 \\
(\text{proton time})^2 &= 3.1111\ (\text{meters})^2
\end{aligned}$$

So time between events in the proton frame equals the square root of this, or 1.764 meters of time.

b. Light travels 60 times as far in one minute as it does in one second. Its speed in meters per minute is therefore:

$$\begin{aligned}
&2.99792458 \times 10^8 \text{ meters/second} \times 60 \text{ seconds/minute} \\
&\qquad = 1.798754748 \times 10^{10} \text{ meters/minute}
\end{aligned}$$

So the distance from Earth to Sun is

$$\frac{1.4960 \times 10^{11} \text{ meters}}{1.798754748 \times 10^{10} \text{ meters/minute}} = 8.3169 \text{ light-minutes}$$

This is the distance between the two events in the Earth frame, measured in light-minutes. The Earth-frame time between the two events is 10 minutes, as stated in the problem.

In the frame traveling with the rock, the two events occur at the same place; the time between the two events in this frame equals the spacetime interval — the proper time — between these events:

$$\begin{aligned}(\text{interval})^2 &= (10 \text{ minutes})^2 - (8.3169 \text{ minutes})^2 \\ &= (100 - 69.1708)\ (\text{minutes})^2 \\ &= 30.8292\ (\text{minutes})^2\end{aligned}$$

The time between events in the rest frame of the rock equals the square root of this, or 5.5524 minutes.

c. The distance between departure from Earth and arrival at Proxima Centauri is 4.3 light-years, as given in the problem. The starship moves at 95 percent light speed, or 0.95 light-years/year. Therefore it takes a time 4.3 light-years/(0.95 light-years/year) = 4.53 years to arrive at Proxima Centauri, as measured in the Earth frame.

Starship time between departure from Earth and arrival at Proxima Centauri equals the interval:

$$\begin{aligned}(\text{interval})^2 &= (4.53 \text{ years})^2 - (4.3 \text{ years})^2 \\ &= (20.52 - 18.49)\ (\text{years})^2 \\ &= 2.03\ (\text{years})^2\end{aligned}$$

The time between events in the rest frame of the starship equals the square root of this, or 1.42 years. Compare with the value 4.53 years as measured in the Earth frame. This example illustrates the famous idea that astronaut wristwatch time — proper time — between two events is less than the time between these events measured by any other observer in relative motion. Travel to stay young! This result comes simply and naturally from the invariance of the interval.

1.5 UNITY OF SPACETIME

time and space: equal footing but distinct nature

When time and space are measured in the same unit — whether meter or second or year — the expression for the square of the spacetime interval between two events takes on a particularly simple form:

$$\begin{aligned}(\text{interval})^2 &= (\text{time separation})^2 - (\text{space separation})^2 \\ &= t^2 - x^2 \qquad \text{[same units for time and space]}\end{aligned}$$

Spacetime is a unity

This formula shows forth the unity of space and time. Impressed by this unity, Einstein's teacher Hermann Minkowski (1864–1909) wrote his famous words, "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a union of the two will preserve an independent reality." Today this union of space and time is called spacetime. Spacetime provides the true theater for

BOX 1-1

PAYOFF OF THE PARABLE

from distance in space to interval in spacetime

DISCUSSION	SURVEYING TOWNSHIP	ANALYZING NATURE
Location marker	Steel stake driven in ground	Collision between two particles Emission of flash from atom Spark jumping from antenna to pen
General name for such a location marker	**Point** or **place**	**Event**
Can its location be staked out for all to see, independent of any scheme of measurement, and independent of all numbers?	Yes	Yes
Simple descriptor of separation between two location markers	**Distance**	**Spacetime interval**
Are there ways directly to measure this separation?	Yes	Yes
With enough markers already staked out, how can we tell someone where we want the next one?	Specify **distances** from other points.	Specify **spacetime intervals** from other events.
Instead of boldly staking out the new marker, or instead of positioning it relative to existing markers, how else can we place the new marker?	By locating point relative to a **reference frame**	By locating event relative to a **reference frame**
Nature of this reference frame?	**Surveyor's grid** yields northward and eastward readings of point (Chapter 1).	**Lattice frame** of rods and clocks yields space and time readings of event (Chapter 2).
Is such a reference frame unique?	No	No
How do two such reference frames differ from one another?	**Tilt** of one surveyor's grid relative to the other	**Uniform velocity** of one frame relative to the other
What are names of two such possible reference frames?	**Daytime grid:** oriented to magnetic north **Nighttime grid:** oriented to North-Star north	**Laboratory frame** **Rocket frame**
What common unit simplifies analysis of the results?	The unit **meter** for both northward and eastward readings	The unit **meter** for both space and time readings
What is the conversion factor from conventional units to meters?	Converting miles to meters: $k = 1609.344$ meters/mile	Converting seconds to meters using the speed of light: $c = 299{,}792{,}458$ meters/second

DISCUSSION	SURVEYING TOWNSHIP	ANALYZING NATURE
For convenience, all measurements are referred to what location?	A common **origin** (center of town)	A common **event** (reference spark)
How do readings for a single marker differ between two reference frames?	Individual northward and eastward readings for one point — for one steel stake — do not have the same values respectively for two surveyors' grids that are tilted relative to one another.	Individual space and time readings for one event — for one spark — do not have the same values respectively for two frames that are in motion relative to one another.
When we change from one marker to two, how do we specify the offset between them in reference-frame language?	**Subtract:** Figure the difference between eastward readings of the two points; also the difference in northward readings.	**Subtract:** Figure the difference between space readings of the two events; also the difference in time readings.
How to figure from offset readings a measure of separation that has the same value whatever the choice of reference frame?	Figure the **distance** between the two points.	Figure the **spacetime interval** between the two events.
Figure how?	$(\text{distance})^2 = \begin{pmatrix}\text{difference in}\\ \text{northward readings}\end{pmatrix}^2 + \begin{pmatrix}\text{difference in}\\ \text{eastward readings}\end{pmatrix}^2$	$(\text{interval})^2 = \begin{pmatrix}\text{difference in}\\ \text{time readings}\end{pmatrix}^2 - \begin{pmatrix}\text{difference in}\\ \text{space readings}\end{pmatrix}^2$
Result of this reckoning?	Distance between points as figured from readings using one surveyor's grid is the **same** as figured from readings using a second surveyor's grid tilted with respect to first grid.	Interval between events as figured from readings using one latticework frame is the **same** as figured from readings using a second frame in steady straight-line motion relative to first frame.
Phrase to summarize this identity of separation as figured in two reference frames?	**Invariance of the distance** between points	**Invariance of the spacetime interval** between events.
Conclusions from this analysis?	(1) Northward and eastward dimensions are part of a single entity: **space.** (2) **Distance** is the simple measure of separation between two **points,** natural because invariant: the same for different surveyor grids.	(1) Space and time dimensions are part of a single entity: **spacetime.** (2) **Spacetime interval** is the simple measure of separation between two **events,** natural because invariant: the same for different reference frames.

every event in the lives of stars, atoms, and people. Space is different for different observers. Time is different for different observers. Spacetime is the same for everyone.

Minkowski's insight is central to the understanding of the physical world. It focuses attention on those quantities, such as spacetime interval, electrical charge, and particle mass, that are the same for all observers in relative motion. It brings out the merely relative character of quantities such as velocity, momentum, energy, separation in time, and separation in space that depend on relative motion of observers.

Difference between time and space

Today we have learned not to overstate Minkowski's argument. It is right to say that time and space are inseparable parts of a larger unity. It is wrong to say that time is identical in quality with space.

Why is it wrong? Is not time measured in meters, just as space is? In relating the positions of two steel stakes driven into the ground, does not the surveyor measure northward and eastward separations, quantities of identical physical character? By analogy, in locating two events is not the observer measuring quantities of the same nature: space and time separations? How else could it be legitimate to treat these quantities on an equal footing, as in the formula for the interval?

Equal footing, yes; same nature, no. There is a minus sign in the formula for the interval squared = (time separation)^2 − (space separation)^2 that no sleight of hand can ever conjure away. This minus sign distinguishes between space and time. No twisting or turning can ever give the same sign to real space and time separations in the expression for the interval.

The invariance of the spacetime interval evidences the unity of space and time while also preserving—in the formula's minus sign—the distinction between the two.

The principles of special relativity are remarkably simple—simpler than the axioms of Euclidean geometry or the principles of operating an automobile. Yet both Euclid and the automobile have been mastered—perhaps with insufficient surprise—by generations of ordinary people. Some of the best minds of the twentieth century struggled with the concepts of relativity, not because nature is obscure, but because (1) people find it difficult to outgrow established ways of looking at nature, and (2) the world of the very fast described by relativity is so far from common experience that everyday happenings are of limited help in developing an intuition for its descriptions.

By now we have won the battle to put relativity in understandable form. The concepts of relativity can now be expressed simply enough to make it easy to think correctly—"to make the bad difficult and the good easy." This leaves only the second difficulty, that of developing intuition—a practiced way of seeing. We understand distance intuitively from everyday experience. Box 1.1 applies our intuition for **distance in space** to help our intuition for **interval in spacetime.**

To put so much into so little, to subsume all of Einstein's teaching on light and motion in the single word *spacetime*, is to cram a wealth of ideas into a small picnic basket that we shall be unpacking throughout the remainder of this book.

REFERENCES

Introductory quote: Richard P. Feynman, *The Character of Physical Law* (MIT Press, Cambridge, Mass., 1967), page 127.

Quote from Minkowski in Section 1.5: H. A. Minkowski, "Space and Time," in H. A. Lorentz *et al.*, *The Principle of Relativity* (Dover Publications, New York, 1952), page 75.

Quote at end of Section 1.5: "to make the bad difficult and the good easy," *"rend le mal difficile et le bien facile."* Einstein, in a similar connection, in a letter to the architect Le Corbusier. Private communication from Le Corbusier.

For an appreciation of Albert Einstein, see John Archibald Wheeler, "Albert Einstein," in *The World Treasury of Physics, Astronomy, and Mathematics,* Timothy Ferris, ed. (Little, Brown, New York, 1991), pages 563–576.

ACKNOWLEDGMENTS

Many students in many classes have read through sequential versions of this text, shared with us their detailed difficulties, and given us advice. We asked students to write down comments, perplexities, and questions as they read and turn in these reading memos for personal response by the teacher. Italicized objections in the text come, in part, from these commentators. Both we who write and you who read are in their debt. Some readers not in classes have also been immensely helpful; among these we especially acknowledge Steven Bartlett. No one could have read the chapters more meticulously than Eric Sheldon, whose wide knowledge has enriched and clarified the presentation. William A. Shurcliff has been immensely inventive in devising new ways of viewing the consequences of relativity; a few of these are specifically acknowledged in later chapters. Electronic-mail courses using this text brought a flood of comments and reading memos from teachers and students around the world. Richard C. Smith originated, organized, and administered these courses, for which we are very grateful. The clarity and simplicity of both the English and the physics were improved by Penny Hull.

Some passages in this text, both brief and extended, have been adapted from the book *A Journey into Gravity and Spacetime* by John Archibald Wheeler (W.H. Freeman, New York, 1990). In turn, certain passages in that book were adapted from earlier drafts of the present text. We have also used passages, logical arguments, and figures from the book *Gravitation* by Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler (W. H. Freeman, New York, 1973).

INTRODUCTION TO THE EXERCISES

Important areas of current research can be analyzed very simply using the theory of relativity. This analysis depends heavily on a physical intuition, which develops with experience. Wide experience is not easy to obtain in the laboratory—simple experiments in relativity are difficult and expensive because the speed of light is so great. As alternatives to experiments, the exercises and problems in this text evoke a wide range of physical consequences of the properties of spacetime. These properties of spacetime recur here over and over again in different contexts:

- paradoxes
- puzzles

- derivations
- technical applications
- experimental results
- estimates
- precise calculations
- philosophical difficulties

The text presents all formal tools necessary to solve these exercises and problems, but intuition — a practiced way of seeing — is best developed without hurry. For this reason we suggest continuing to do more and more of these exercises in relativity after you have moved on to material outside this book. The mathematical manipulations in the exercises and problems are very brief: only a few answers take more than five lines to write down. On the other hand, the exercises require some "rumination time."

In some chapters, exercises are divided into two categories, Practice and Problems. The Practice exercises help you to get used to ideas in the text. The Problems apply these ideas to physical systems, thought experiments, and paradoxes.

WHEELER'S FIRST MORAL PRINCIPLE: *Never make a calculation until you know the answer.* Make an estimate before every calculation, try a simple physical argument (symmetry! invariance! conservation!) before every derivation, guess the answer to every paradox and puzzle. Courage: No one else needs to know what the guess is. Therefore make it quickly, by instinct. A right guess reinforces this instinct. A wrong guess brings the refreshment of surprise. In either case life as a spacetime expert, however long, is more fun!

CHAPTER 1 EXERCISES

PRACTICE

1-1 comparing speeds

Compare the speeds of an automobile, a jet plane, an Earth satellite, Earth in its orbit around Sun, and a pulse of light. Do this by comparing the relative distance each travels in a fixed time. Arbitrarily choose the fixed time to give convenient distances. A car driving at the USA speed limit of 65 miles/hour (105 kilometers/hour) covers 1 meter of distance in about 35 milliseconds $= 35 \times 10^{-3}$ second.

a How far does a commercial jetliner go in 35 milliseconds? (speed: 650 miles/hour = 1046 kilometers/hour)

b How far does an Earth satellite go in 35 milliseconds? (speed: 17,000 miles/hour $\approx$ 27,350 kilometers/hour)

c How far does Earth travel in its orbit around Sun in 35 milliseconds? (speed: 30 kilometers/second)

d How far does a light pulse go in a vacuum in 35 milliseconds? (speed: 3×10^8 meters /second). This distance is roughly how many times the distance from Boston to San Francisco (5000 kilometers)?

1-2 images from Neptune

At 9:00 P.M. Pacific Daylight Time on August 24, 1989, the planetary probe *Voyager II* passed by the planet Neptune. Images of the planet were coded and transmitted to Earth by microwave relay.

It took 4 hours and 6 minutes for this microwave signal to travel from Neptune to Earth. Microwaves (electromagnetic radiation, like light, but of frequency lower than that of visible light), when propagating through interplanetary space, move at the "standard" light speed of one meter of distance in one meter of light-travel time, or 299,792,458 meters/second. In the following, neglect any relative motion among Earth, Neptune, and *Voyager II.*

a Calculate the distance between Earth and Neptune at fly-by in units of minutes, seconds, years, meters, and kilometers.

b Calculate the time the microwave signal takes to reach Earth. Use the same units as in part **a**.

1-3 units of spacetime

Light moves at a speed of 3.0×10^8 meters/second. One mile is approximately equal to 1600 meters. One furlong is approximately equal to 200 meters.

a How many meters of time in one day?
b How many seconds of distance in one mile?
c How many hours of distance in one furlong?
d How many weeks of distance in one light-year?
e How many furlongs of time in one hour?

1-4 time stretching and the spacetime interval

A rocket clock emits two flashes of light and the rocket observer records the time lapse (in seconds) between these two flashes. The laboratory observer records the time separation (in seconds) and space separation (in light-seconds) between the same pair of flashes. The results for both laboratory and rocket observers are recorded in the first line of the table.

Now a clock in a different rocket, moving at a different speed with respect to the laboratory, emits a different pair of flashes. The set of laboratory and rocket space and time separations are recorded on the second line of the table. And so on. Complete the table.

EXERCISE 1-4

SPACE AND TIME SEPARATIONS

	Rocket time lapse (seconds)	Laboratory time lapse (seconds)	Laboratory distance (light-seconds)
Example	20	29	21
a	?	10.72	5.95
b	20	?	99
c	66.8	72.9	?
d	?	8.34	6.58
e	21	22	?

1-5 where and when?

Two firecrackers explode at the same place in the laboratory and are separated by a time of 3 years as measured on a laboratory clock.

a What is the spatial distance between these two events in a rocket in which the events are separated in time by 5 years as measured on rocket clocks?

b What is the relative speed of the rocket and laboratory frames?

1-6 mapmaking in space

The table shows distances between cities. The units are kilometers. Assume all cities lie on the same flat plane.

a Use a ruler and a compass (the kind of compass that makes circles) to construct a map of these cities. Choose a convenient scale, such as one centimeter on the map corresponds to ten kilometers on Earth.

Discussion: How to start? With three arbitrary decisions! (1) Choose any city to be at the center of the map. (2) Choose any second city to be "due north" — that is, along any arbitrary direction you select. (3) Even with these choices, there are two places you can locate the third city; choose either of these two places arbitrarily.

b If you rotate the completed map in its own plane — for example, turning it while keeping it flat on the table — does the resulting map also satisfy the distance entries above?

c Hold up your map between you and a light, with the marks on the side of the paper facing the

EXERCISE 1-6

DISTANCES BETWEEN CITIES

Distance to city / from city	A	B	C	D	E	F	G	H
A	0	20.0	28.3	28.3	28.3	20.0	28.3	44.7
B		0	20.0	20.0	44.7	40.0	44.7	40.0
C			0	40.0	40.0	44.7	56.6	60.0
D				0	56.6	44.7	40.0	20.0
E					0	20.0	40.0	72.1
F						0	20.0	56.6
G							0	44.7
H								0

light. Does the map you see from the back also satisfy the table entries?

Discussion: In this exercise you use a table consisting only of distances between pairs of cities to construct a map of these cities from the point of view of a surveyor using a given direction for north. In Exercise 5-3 you use a table consisting only of spacetime intervals between pairs of events to draw a "spacetime map" of these events from the point of view of one free-float observer. Exercise 1-7 previews this kind of spacetime map.

1-7 spacetime map

The laboratory space and time measurements of events 1 through 5 are plotted in the figure. Compute the value of the spacetime interval

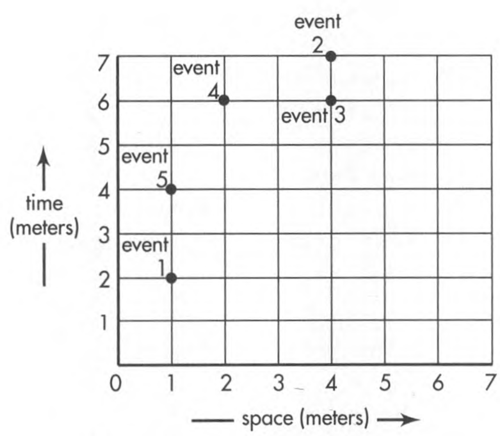
a between event 1 and event 2.

b between event 1 and event 3.

c between event 1 and event 4.

d between event 1 and event 5.

e A rocket moves with constant velocity from event 1 to event 2. That is, events 1 and 2 occur at the same place in this rocket frame. What time lapse is recorded on the rocket clock between these two events?



EXERCISE 1-7. *Spacetime map of some events.*

PROBLEMS

1-8 size of a computer

In one second some desktop computers can carry out one million instructions in sequence. One instruction might be, for instance, multiplying two numbers together. In technical jargon, such a computer operates at "one megaflop." Assume that carrying out one instruction requires transmission of data from the memory (where data is stored) to the processor (where the computation is carried out) and transmission of the result back to the memory for storage.

a What is the maximum average distance between memory and processor in a "one-megaflop" computer? Is this maximum distance increased or decreased if the signal travels through conductors at one half the speed of light in a vacuum?

b Computers are now becoming available that operate at "one gigaflop," that is, they carry out 10^9 sequential instructions per second. What is the maximum average distance between memory and processor in a "one-gigaflop" machine?

c Estimate the overall maximum size of a "oneteraflop" machine, that is, a computer that can carry out 10^{12} sequential instructions per second.

d Discussion question: In contrast with most current personal computers, a "parallel processing" computer contains several or many processors that work together on a computing task. One might think that a machine with 10,000 processors would complete a given computation task in 1/10,000 the time. However, many computational problems cannot be divided up in this way, and in any case some fraction of the computing capacity must be devoted to coordinating the team of processors. What limits on physical size does the speed of light impose on a parallel processing computer?

1-9 trips to Andromeda by rocket

The Andromeda galaxy is approximately two million light-years distant from Earth as measured in the Earth-linked frame. Is it possible for you to travel from Earth to Andromeda in your lifetime? Sneak up on the answer to this question by considering a series of trips from Earth to Andromeda, each one faster than the one before. For simplicity, assume the Earth-Andromeda distance to be exactly two million light-years in the Earth frame, treat Earth and Andromeda as points, and neglect any relative motion between Earth and Andromeda.

a TRIP 1. Your one-way trip takes a time 2.01×10^6 years (measured in the Earth-linked frame) to cover the distance of 2.00×10^6 light-years. How long does the trip last as measured in your rocket frame?

b What is your rocket speed on Trip 1 as measured in the Earth-linked frame? Express this speed as a decimal fraction of the speed of light. Call this fraction, $v = v_{conv}/c$, where v_{conv} is speed in conventional units, such as meters/second. **Discussion:** If your rocket moves at half the speed of light, it takes

4×10^6 years to cover the distance 2×10^6 light-years. In this case

$$v = \frac{2 \times 10^6 \text{ light-years}}{4 \times 10^6 \text{ years}} = \frac{1}{2}$$

Therefore . . .

c TRIP 2. Your one-way Earth-Andromeda trip takes 2.001×10^6 years as measured in the Earth-linked frame. How long does the trip last as measured in your rocket frame? What is your rocket speed for Trip 2, expressed as a decimal fraction of the speed of light?

d TRIP 3. Now set the rocket time for the one-way trip to 20 years, which is all the time you want to spend getting to Andromeda. In this case, what is your speed as a decimal fraction of the speed of light?

Discussion: Solutions to many exercises in this text are simplified by using the following approximation, which is the first two terms in the binomial expansion

$$(1 + z)^n \approx 1 + nz \qquad \text{if} \qquad |z| << 1$$

Here n can be positive or negative, a fraction or an integer; z can be positive or negative, as long as its magnitude is very much smaller than unity. This approximation can be used twice in the solution to part **d**.

1-10 trip to Andromeda by Transporter

In the *Star Trek* series a so-called Transporter is used to "beam" people and their equipment from a starship to the surface of nearby planets and back. The Transporter mechanism is not explained, but it appears to work only locally. (If it could transport to remote locations, why bother with the starship at all?) Assume that one thousand years from now a Transporter exists that reduces people and things to data (elementary bits of information) and transmits the data by light or radio signal to remote locations. There a Receiver uses the data to reassemble travelers and their equipment out of local raw materials.

One of your descendants, named Samantha, is the first "transporternaut" to be beamed from Earth to the planet Zircon orbiting a star in the Andromeda Nebula, two million light-years from Earth. Neglect any relative motion between Earth and Zircon, and assume: (1) transmission produces a Samantha identical to the original in every respect (except that she is 2 million light-years from home!), and (2) the time required for disassembling Samantha on Earth and reassembling her on Zircon is negligible as measured in the common rest frame of Transporter and Receiver.

a How much does Samantha age during her outward trip to Zircon?

b Samantha collects samples and makes observations of the Zirconian civilization for one Earth-year, then beams back to Earth. How much has Samantha aged during her entire trip?

c How much older is Earth and its civilization when Samantha returns?

d Earth has been taken over by a tyrant, who wishes to invade Zircon. He sends one warrior and has him duplicated into attack battalions at the Receiver end. How long will the Earth tyrant have to wait to discover whether his ambition has been satisfied?

e A second transporternaut is beamed to a much more remote galaxy that is moving away from Earth at 87 percent of the speed of light. This time, too, the traveler stays in the remote galaxy for one year *as measured by clocks moving with the galaxy* before returning to Earth by Transporter. How much has the transporternaut aged when she arrives back at Earth? (Careful!)

1-11 time stretching with muons

At heights of 10 to 60 kilometers above Earth, cosmic rays continually strike nuclei of oxygen and nitrogen atoms and produce muons (muons: elementary particles of mass equal to 207 electron masses produced in some nuclear reactions). Some of the muons move vertically downward with a speed nearly that of light. Follow one of the muons on its way down. In a given sample of muons, half of them decay to other elementary particles in 1.5 microseconds (1.5×10^{-6} seconds), measured with respect to a reference frame in which they are at rest. Half of the remainder decay in the next 1.5 microseconds, and so on. Analyze the results of this decay as observed in two different frames. Idealize the rather complicated actual experiment to the following roughly equivalent situation: All the muons are produced at the same height (60 kilometers); all have the same speed; all travel straight down; none are lost to collisions with air molecules on the way down.

a Approximately how long a time will it take these muons to reach the surface of Earth, as measured in the Earth frame?

b If the decay time were the same for Earth observers as for an observer traveling with the muons, approximately how many half-lives would have passed? Therefore what fraction of those created at a height of 60 kilometers would remain when they reached sea level on Earth? You may express your answer as a power of the fraction 1/2.

c An experiment determines that the fraction 1/8 of the muons reaches sea level. Call the rest frame of the muons the rocket frame. In this rocket frame, how many half-lives have passed between creation of a given muon and its arrival as a survivor at sea level?

d *In the rocket frame,* what is the space separation between birth of a survivor muon and its arrival at the surface of Earth? (Careful!)

e From the rocket space and time separations, find the value of the spacetime interval between the birth event and the arrival event for a single surviving muon.

Reference: Nalini Easwar and Douglas A. MacIntire, *American Journal of Physics,* Volume 59, pages 589–592 (July 1991).

EXERCISE 1-12

TIME STRETCHING WITH π^+-MESONS

Particle	Time for half to decay (measured in rest frame)	"Characteristic distance" (speed of light multiplied by foregoing time)
muon (207 times electron mass)	1.5×10^{-6} second	450 meters
π^+-meson (273 times electron mass)	18×10^{-9} second	5.4 meters

1-12 time stretching with π^+-mesons

Laboratory experiments on particle decay are much more conveniently done with π^+-mesons (pi-plus mesons) than with μ-mesons, as is seen in the table.

In a given sample of π^+-mesons half will decay to other elementary particles in 18 nanoseconds (18×10^{-9} seconds) measured in a reference frame in which the π^+-mesons are at rest. Half of the remainder will decay in the next 18 nanoseconds, and so on.

a In a particle accelerator π^+-mesons are produced when a proton beam strikes an aluminum target inside the accelerator. Mesons leave this target with nearly the speed of light. If there were no time stretching and if no mesons were removed from the resulting beam by collisions, what would be the greatest distance from the target at which half of the mesons would remain undecayed?

b The π^+-mesons of interest in a particular experiment have a speed 0.9978 that of light. By what factor is the predicted distance from the target for half-decay increased by time dilation over the previous prediction — that is, by what factor does this dilation effect allow one to increase the separation between the detecting equipment and target?