# CHAPTER 2

# FLOATING FREE

> **A**t that moment there came to me the happiest thought of my life . . . *for an observer falling freely from the roof of a house no gravitational field exists during his fall . . .*
>
> Albert Einstein

## 2.1 FLOATING TO MOON

### will the astronaut stand on the floor — or float?

Less than a month after the surrender at Appomattox ended the American Civil War (1861 – 1865), the French author Jules Verne began writing *A Trip From the Earth to the Moon* and *A Trip Around the Moon.* Eminent American cannon designers, so the story goes, cast a great cannon in a pit, with cannon muzzle pointing skyward. From this cannon they fire a ten-ton projectile containing three men and several animals (Figure 2-1).

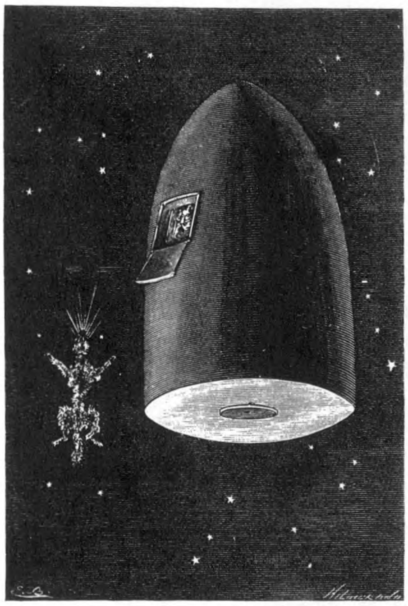
Jules Verne:
Passenger stands on floor

As the projectile coasts outward in unpowered flight toward Moon, Verne says, its passengers walk normally inside the projectile on the end nearer Earth (Figure 2-2). As the trip continues, passengers find themselves pressed less and less against the floor of the spaceship until finally, at the point where Earth and Moon exert equal but opposite gravitational attraction, passengers float free of the floor. Later, as the ship nears Moon, they walk around once again — according to Verne — but now against the end of the spaceship nearer Moon.

Early in the coasting portion of the trip a dog on the ship dies from injuries sustained at takeoff. Passengers dispose of its remains through a door in the spaceship, only to find the body floating outside the window during the entire trip (Figure 2-1).

Paradox of passenger and dog

This story leads to a paradox whose resolution is of crucial importance to relativity. Verne thought it reasonable that Earth's gravitational attraction would keep a passenger pressed against the Earth end of the spaceship during the early part of the trip. He also thought it reasonable that the dog should remain next to the ship, since both ship and dog independently follow the same path through space. But since the dog floats outside the spaceship during the entire trip, why doesn't the passenger float around inside the spaceship? If the ship were sawed in half would the passenger, now "outside," float free of the floor?



IT WAS THE BODY OF SATELLITE.

**FIGURE 2-1.** ***Illustration from an early edition of*** **A Trip Around the Moon.** *Satellite is the name of the unfortunate dog.*

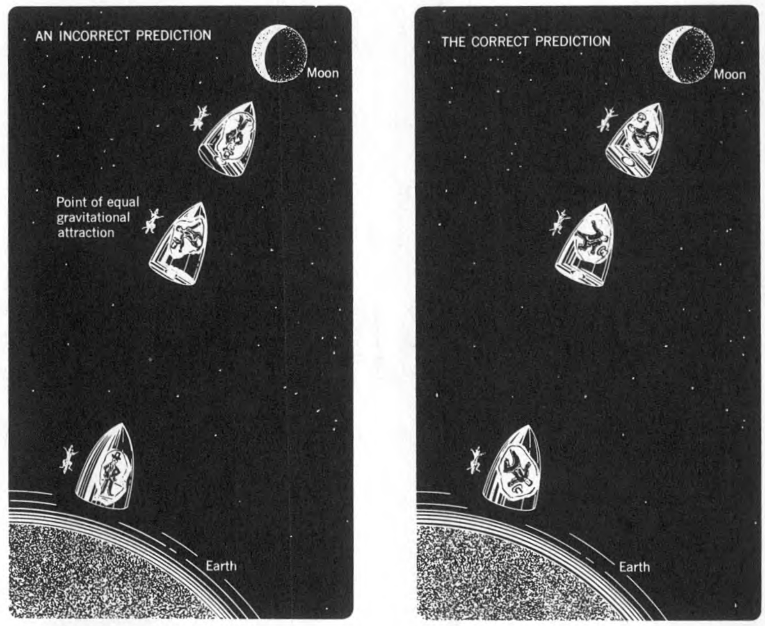



**FIGURE 2-2.** ***Incorrect prediction:*** *Jules Verne believed that a passenger inside a free projectile would stand against the end of the projectile nearest Earth or Moon, whichever had greater gravitational attraction — but that the dog would float along beside the projectile for the entire trip.* ***Correct prediction:*** *Verne was right about the dog, but a passenger also floats with respect to the free projectile during the entire trip.*

Reality:
Passenger floats in spaceship

Our experience with actual space flights enables us to resolve this paradox (Figure 2-2). Jules Verne was wrong about the passenger's motion inside the unpowered spaceship. Like the dog outside, the passenger inside independently follows the same path through space as the spaceship itself. Therefore he floats freely relative to the ship during the entire trip (after the initial boost inside the cannon barrel). True: Earth's gravity acts on the passenger. But it also acts on the spaceship. In fact, with respect to Earth, gravitational acceleration of the spaceship just equals gravitational acceleration of the passenger. Because of this equality, there is no *relative* acceleration between passenger and spaceship. Thus the spaceship serves as a **reference frame** relative to which the passenger does not experience any acceleration.

To say that acceleration of the passenger relative to the unpowered spaceship equals zero is *not* to say that his velocity relative to it necessarily also equals zero. He may jump from the floor or spring from the side — in which case he hurtles across the spaceship and strikes the opposite wall. However, when he floats with zero initial velocity relative to the ship the situation is particularly interesting, for he will also float with zero velocity relative to it at all later times. He and the ship follow identical paths through space. How remarkable that the passenger, who cannot see outside, nevertheless moves on this deterministic orbit! Without a way to control his motion and even with his eyes closed he will not touch the wall. How could one do better at eliminating detectable gravitational influences?

# 2.2 THE INERTIAL (FREE-FLOAT) FRAME

## goodbye to the "force of gravity"

It is easy to talk about the simplicity of motion in a spaceship. It is hard to think of conditions being equally simple on the surface of Earth (Figure 2-3). The reason for

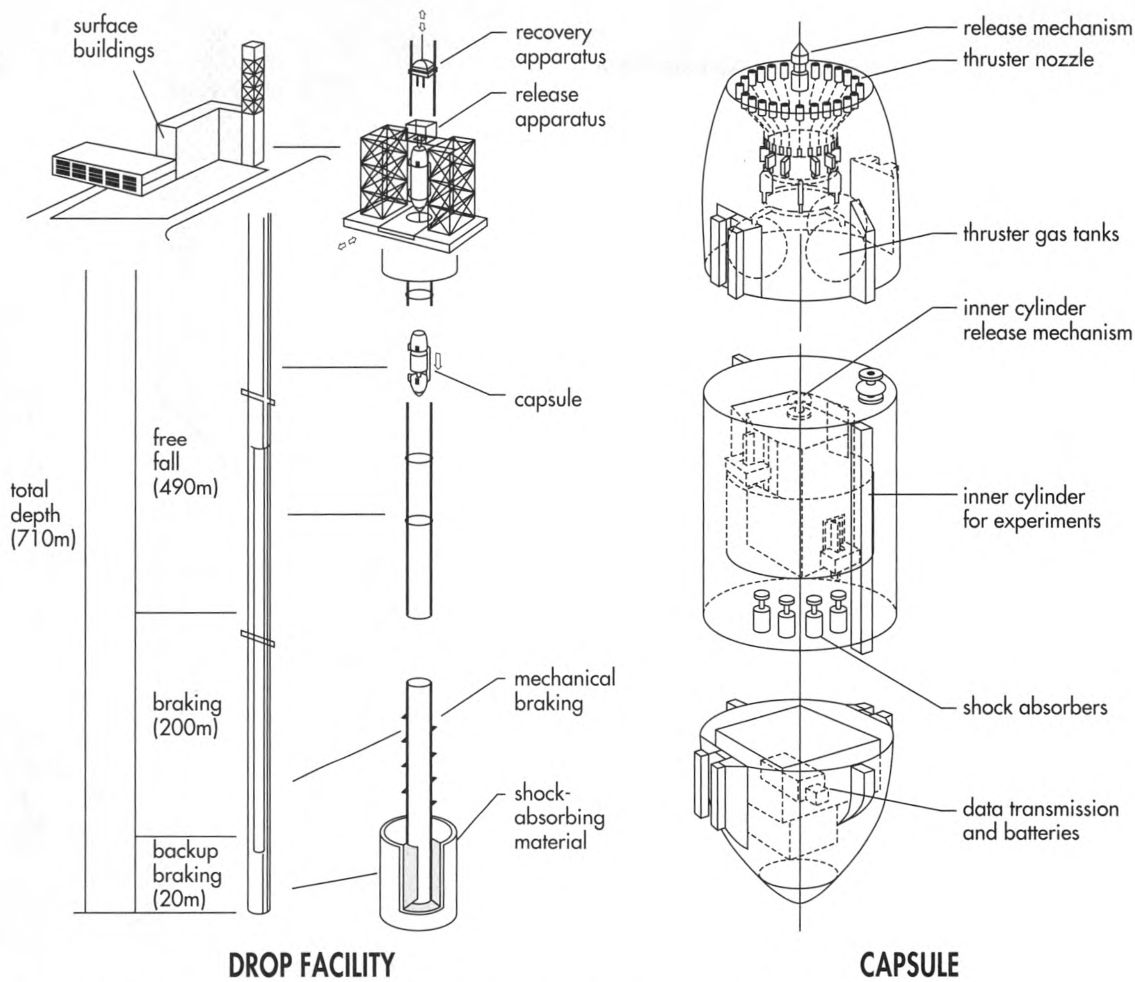



**FIGURE 2-3.** ***The Japan Microgravity Center (JAMIC) installed in an abandoned coal mine 710 meters deep in the small town of Kamisunagawa on the northern island of Hokkaido, Japan.*** *The capsule carrying the experimental apparatus provides a free-float frame for 10 seconds as it falls 490 meters through a vertical tube, achieving a maximum velocity of nearly 100 meters/second. It is guided by two contact-free magnetic suspensions along the tube. The vertical tube is not evacuated; downward-thrusting gas jets on the capsule compensate for air drag as the capsule drops. The capsule is slowed down in an additional distance of 200 meters near the bottom of the tube by air resistance after thrusters are turned off, followed by mechanical braking. Twenty meters of cushioning material at the very bottom of the tube provide emergency stopping. The falling capsule is nearly 8 meters long and nearly 2 meters in diameter with a mass of 5000 kilograms, including 1000 kilograms of experimental equipment contained in an inner cylinder 1.3 meters in diameter and 1.8 meters long. The space between capsule and experimental cylinder is evacuated. The inner experimental cylinder is released just before the outer capsule itself. Optical monitoring of the vertical position of the inner cylinder triggers downward-pushing thrusters as needed to overcome air resistance. Thus the experimental cylinder itself acts as an internal "conscience," ensuring that the capsule takes the same course that it would have taken had both resistance and thrust been absent. The result? A nearly free-float frame, with a maximum acceleration of* $1.0 \times 10^{-4}$ g *in the experimental capsule, where* g *is the acceleration of gravity at Earth's surface. Experiments carried out in this facility benefit from conditions of "no air pressure, no heat convection, no floating or sinking buoyancy, no resistance to motion," as well as much lower cost and less environmental damage than those involved in launching and monitoring an Earth satellite. The facility is designed to carry out 400 drops per year, with experiments such as forming large superconducting crystals, creating alloys of materials that do not normally mix, studying transitions between gas and liquid phases, and burning under zero-g. (See also Figure 9-2.)*

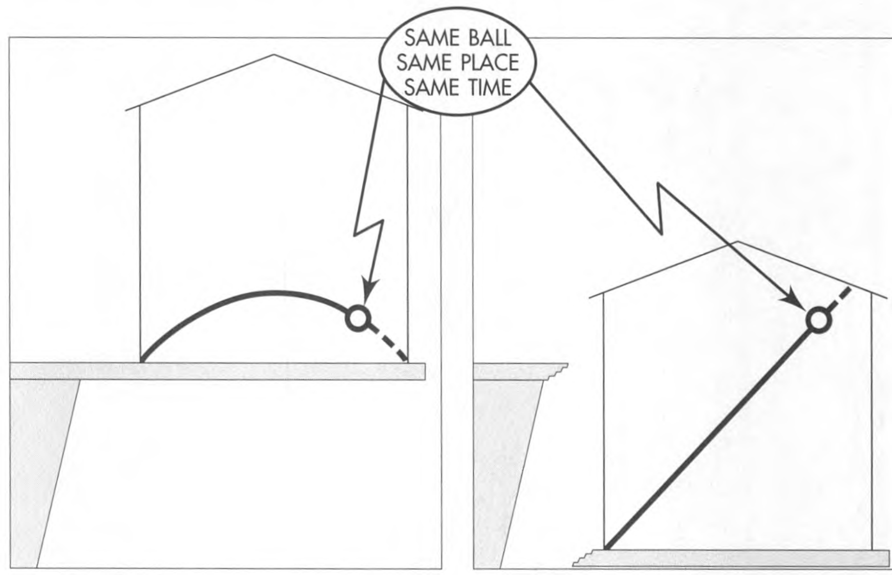



FIGURE 2-4. *Illusion and Reality. The same ball thrown from the same corner of the same room in the same direction with the same speed is seen to undergo very different motions depending on whether it is recorded by an observer with a floor pushing up against his feet or by an observer in "free fall" ("free float") in a house sawed free from the cliff. In both descriptions the ball arrives at the same place—relative to Mother Earth—at the same instant. Let each ball squirt a jet of ink on the wall we are looking at. The resulting record is as crisp for the arc as for the straight line. Is the arc real and the straight line illusion? Or is the straight line real and the arc illusion? Einstein tells us that the two ink trails are equally valid. We have only to be honest and say whether the house, the wall, and the describer of the motion are in free float or whether the describer is continually being driven away from a condition of free float by a push against his feet. Einstein also tells us that physics always looks simplest in a free-float frame. Finally, he tells us that every truly local manifestation of "gravity" can be eliminated by observing motion from a frame of reference that is in free float.*

concern is not far to seek. We experience it every day, every minute, every second. We call it gravity. It shows in the arc of a ball tossed across the room (Figure 2-4, left). How can anyone confront a mathematical curve like that arc and not be trapped again in that tortuous trail of thought that led from ancient Greeks to Galileo to Newton? They thought of gravity as a force acting through space, as something mysterious, as something that had to be "explained."

Einstein put forward a revolutionary new idea. Eliminate gravity!

Where lies the cause of the curved path of the ball? Is it the ball? Is it some mysterious "force of gravity"? Neither, Einstein tells us. It is the fault of the viewers—and the fault of the floor that forces us away from the natural state of motion: the state of **free fall,** or better put, **free float.** Remove the floor and our motion immediately becomes natural, effortless, free from gravitational effects.

Concept of free-float frame

Let the room be cut loose at the moment we throw the ball slantwise upward from the west side at floor level (Figure 2-4, right). The ball has the same motion as it did before. However, the motion looks different. It looks different because we who look at it are in a different frame of reference. We are in a **free-float frame.** In this free-float frame the ball has straight-line motion. What could be simpler?

Even when the room was not cut away from the cliff, the floor did not affect the midair flight of the ball. But the floor did affect us who watched the flight. The floor forced us away from our natural motion, the motion of free fall (free float). We blamed the curved path of the ball on the "force of gravity" acting on the *ball.* Instead we should have blamed the floor for its force acting on *us.* Better yet, get rid of the floor by cutting the house away from the cliff. Then our point of view becomes the natural one: We enter a free-float frame. In our free-float frame the ball flies straight.

*What's the fault of the force on my feet?*
*What pushes my feet down on the floor?*
*Says Newton, the fault's at Earth's core.*
*Einstein says, the fault's with the floor;*
*Remove that and gravity's beat!*

—Frances Towne Ruml

How could humankind have lived so many centuries without realizing that the "arc" is an unnecessary distraction, that the idea of local "gravity" is superfluous—the fault of the observer for not arranging to look at matters from a condition of free float?

Even today we recoil instinctively from the experience of free float. We and a companion ride in the falling room, which does not crash on the ground but drops into a long vertical tunnel dug for that purpose along the north–south axis of Earth. Our companion is so filled with consternation that he takes no interest in our experimental findings about free float. He grips the door jamb in terror. "We're falling!" he cries out. His fear turns to astonishment when we tell him not to worry.

"A shaft has been sunk through Earth," we tell him. "It's not the fall that hurts anyone but what stops the fall. All obstacles have been removed from our way, including air. Free fall," we assure him, "is the safest condition there is. That's why we call it free float."

"You may call it float," he says, "but I still call it fall."

**Free-float through Earth**

"Right now that way of speaking may seem reasonable," we reply, "but after we pass the center of Earth and start approaching the opposite surface, won't the word 'fall' seem rather out of place? Might you not then prefer the word 'float'?" And with "float" our companion at last is happy.

What do we both see? Weightlessness. Free float. Motion in a straight line and at uniform speed for marbles, pennies, keys, and balls in free motion in any direction within our traveling home. No jolts. No shudders. No shakes at any point in all the long journey from one side of Earth to the other.

For our ancestors, travel into space was a dream beyond realization. Equally beyond our reach today is the dream of a house floating along a tunnel through Earth, but this dream nonetheless illuminates the simplicity of motion in a free-float frame. Given the necessary conditions, nothing that we observe inside our traveling room gives us the slightest possibility of discriminating among different free-float frames: one just above Earth's surface, a second passing through Earth's interior, a third in the uttermost reaches of space. Floating inside any of them we find no evidence whatever for the presence of "gravity."

*Wait a minute! If the idea of local "gravity" is unnecessary, why does my pencil begin to fall when I hold it in the air and let go? If there is no gravity, my pencil should remain at rest.*

And so it does remain at rest—as observed from a free-float frame! The natural motion of your pencil is to remain at rest or to move with constant velocity in a free-float frame. So it is not helpful to ask: "Why does the pencil begin to fall when I let go?" A more helpful question: "Before I let go, why must I apply an upward force to keep the pencil at rest?" Answer: Because you are making observations from an unnatural frame: one held fixed at the surface of Earth. Remove that fixed hold by dropping your room off a cliff. Then for you "gravity" disappears. For you, no force is required to keep the pencil at rest in your free-float frame.

# 2.3 LOCAL CHARACTER OF FREE-FLOAT FRAME

### tidal effects intrude in larger domains

First to strike us about the concept free float has been its paradoxical character. As a first step to explaining gravity Einstein got rid of gravity. There is no evidence of gravity in the freely falling house.

Well, *almost* no evidence. The second feature of free float is its local character. Riding in a very small spaceship (Figure 2-5, left) we find no evidence of gravity. But the enclosure in which we ride — falling near Earth or plunging through Earth — cannot be too large or fall for too long a time without some unavoidable relative changes in motion being detected between particles in the enclosure. Why? Because widely separated particles within a large enclosed space are differently affected by the nonuniform gravitational field of Earth, to use the Newtonian way of speaking. For example, two particles released side by side are both attracted toward the center of Earth, so they move closer together as measured inside a falling long narrow *horizontal* railway coach (Figure 2-5, center). This has nothing to do with "gravitational attraction" between the particles, which is entirely negligible.

Earth's pull nonuniform: Large spaceship not a free-float frame

As another example, think of two particles released far apart vertically but directly above one another in a long narrow *vertical* falling railway coach (Figure 2-5, right). This time their gravitational accelerations toward Earth are in the same direction,

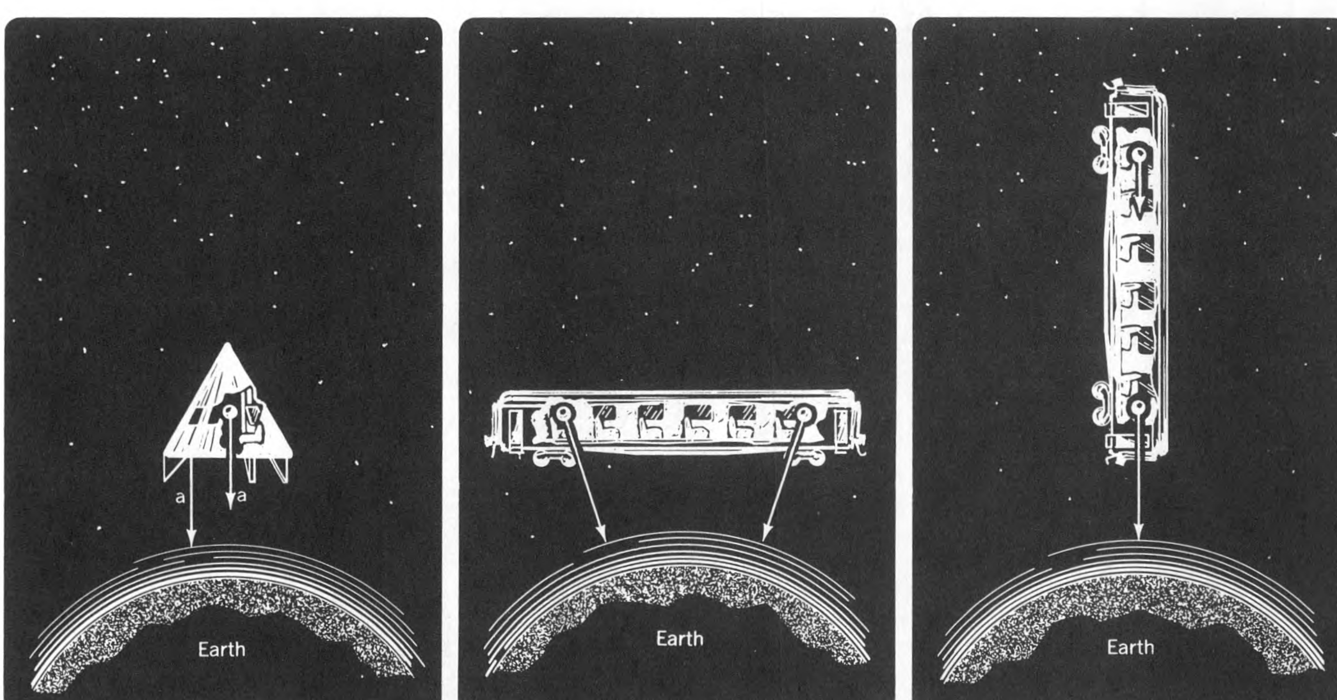



FIGURE 2-5. *Three vehicles in free fall near Earth: small space capsule, Einstein's old-fashioned railway coach in free fall in a* horizontal *orientation, and another railway coach in* vertical *orientation.*

according to the Newtonian analysis. However, the particle nearer Earth is more strongly attracted to Earth and slowly leaves the other behind: the two particles move farther apart as the coach falls. Conclusion: the large enclosure is not a free-float frame.

**Free-float frame is local**

Even a small room fails to qualify as free-float when we sample it over a long enough time. In the 42 minutes it takes our small room to fall through the tunnel from North Pole to South Pole, we notice relative motion between test particles released initially from rest at opposite sides of the room.

Now, we want the laws of motion to look simple in our floating room. Therefore we want to eliminate all relative accelerations produced by external causes. "Eliminate" means to reduce these accelerations below the limit of detection so that they do not interfere with more important accelerations we wish to study, such as those produced when two particles collide. We eliminate the problem by choosing a room that is sufficiently small. Smaller room? Smaller relative accelerations of objects at different points in the room!

Let someone have instruments for detection of relative accelerations with any given degree of sensitivity. No matter how fine that sensitivity, the room can always be made so small that these perturbing relative accelerations are too small to be detectable. Within these limits of sensitivity our room is a free-float frame. "Official" names for such a frame are the **inertial reference frame** and the **Lorentz reference frame.** Here, however, we often use the name **free-float frame,** which we find more descriptive. These are all names for the same thing.

**Free-float (inertial) frame formally defined**

> **A reference frame is said to be an "inertial" or "free-float" or "Lorentz" reference frame in a certain region of space and time when, throughout that region of spacetime — and within some specified accuracy — every free test particle initially at rest with respect to that frame remains at rest, and every free test particle initially in motion with respect to that frame continues its motion without change in speed or in direction.**

Wonder of wonders! This test can be carried out entirely within the free-float frame. The observer need not look out of the room or refer to any measurements made external to the room. A free-float frame is "local" in the sense that it is limited in space and time — and also "local" in the sense that its free-float character can be determined from within, locally.

Sir Isaac Newton stated his First Law of Motion this way: "Every body perseveres in its state of rest, or of uniform motion in a right [straight] line, unless it is compelled to change that state by forces impressed upon it." For Newton, **inertia** was a property of objects that described their tendency to maintain their state of motion, whether of rest or constant velocity. For him, objects obeyed the "Law of Inertia." Here we have turned the "Law of Inertia" around: Before we certify a reference frame to be inertial, we *require* observers in that frame to demonstrate that every free particle maintains its initial state of motion or rest. Then Newton's First Law of Motion *defines* a reference frame — an arena or playing field — in which one can study the motion of objects and draft the laws of their motion.

*When is the room, the spaceship, or any other vehicle small enough to be called a local free-float frame? Or when is the relative acceleration of two free particles placed at opposite ends of the vehicle too slight to be detected?*

"Local" is a tricky word. For example, drop the old-fashioned 20-meter-long railway coach in a horizontal orientation from rest at a height of 315 meters onto the surface of Earth (Figure 2-5, center). Time from release to impact equals 8 seconds, or 2400 million meters of light-travel time. At the same instant you drop the coach, release tiny ball bearings from rest — and in midair — at opposite ends of the coach.

## BOX 2-1

# THE TIDE-DRIVING POWER OF MOON AND SUN

**Note:** *Neither astronomers nor newspapers say "the Venus" or "the Mars." All say simply "Venus" or "Mars." Astronomers follow the same snappy practice for Earth, Moon, and Sun. More and more of the rest of the world now follows — as do we in this book — the recommendations of the International Astronomical Union.*

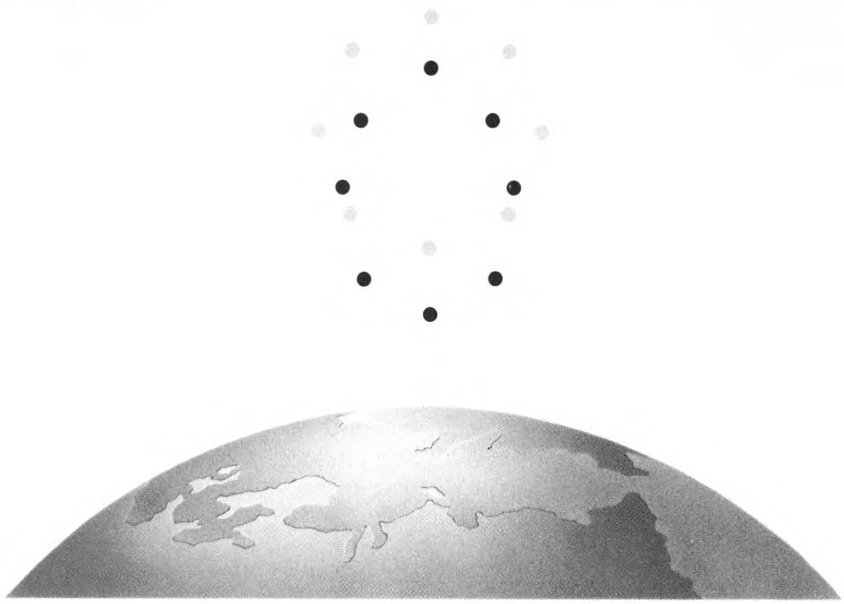
The ocean's rise and fall in a never-ending rhythmic cycle bears witness to the tide-driving power of Moon and Sun. In principle those influences are no different from those that cause relative motion of free particles in the vicinity of Earth. In a free-float frame near Earth, particles separated vertically *increase* their separation with time; particles separated horizontally *decrease* their separation with time (Figure 2-5). More generally, a thin spattering of free-float test masses, spherical in pattern, gradually becomes egg-shaped, with the long axis vertical. Test masses nearer Earth, more strongly attracted than the average, move downward to form the lower bulge. Similarly, test masses farther from Earth, less strongly attracted than the average, lag behind to form the upper bulge.

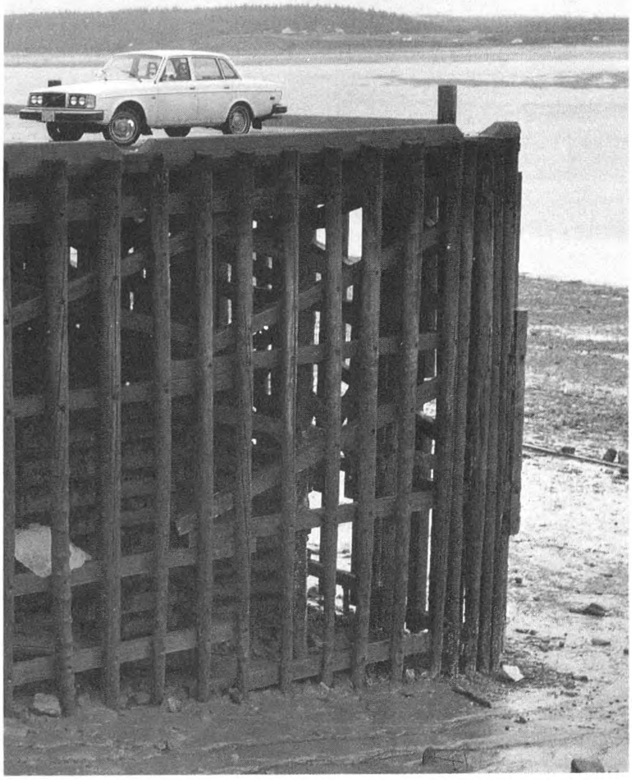
By like action Moon, acting on the waters of Earth — floating free in space — would draw them out into an egg-shaped pattern if there were water everywhere, water of uniform depth. There isn't. The narrow Straits of Gibraltar almost cut off the Mediterranean from the open ocean, and almost kill all tides in it. Therefore it is no wonder that Galileo Galilei, although a great pioneer in the study of gravity, did not take the tides as seriously as the more widely traveled Johannes Kepler, an expert on the motion of Moon and the planets. Of Kepler, Galileo even said, "More than other people he was a person of independent genius . . . [but he] later pricked up his ears and became interested in the action of the moon on the water, and in other occult phenomena, and similar childishness."

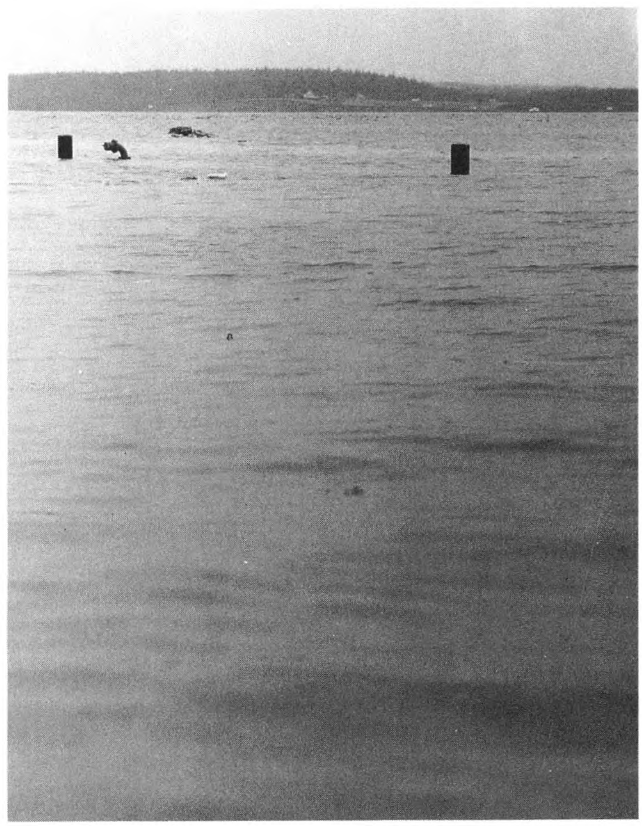
Foolishness indeed, it must have seemed, to assign to the tiny tides of the Mediterranean an explanation so cosmic as Moon. But mariners in northern waters face destruction unless they track the tides. For good reason they remember that Moon reaches its summit overhead an average 50.47 minutes later each day. Their own bitter experience tells them that, of the two high tides a day — *two* because there are two projections on an egg — each also comes about 50 minutes later than it did the day before.

Geography makes Mediterranean tides minuscule. Geography also makes tides in the Gulf of Maine and Bay of Fundy the highest in the world. How come? Resonance! The Bay of Fundy and the Gulf of Maine make together a great bathtub in which water sloshes back and forth with a natural period of 13 hours, near to the 12.4-hour timing of Moon's tide-driving power — and to the 12-hour timing of Sun's influence. Build a big power-producing dam in the upper reaches of the Bay of Fundy? Shorten the length of the bathtub? Decrease the slosh time from 13 hours to exact resonance with Moon? Then get one-foot higher tides along the Maine coast!

Want to see the highest tides in the Bay of Fundy? Then choose your visit according to these rules: (1) Come in summer, when this northern body of water tilts most strongly toward Moon. (2) Come when Moon, in its elliptic orbit, is closest to Earth — roughly 10 percent closer than its most distant point, yielding roughly 35 percent greater tide-producing power. (3) Take into account the tide-producing power of Sun, about 45 percent as great as that of Moon. Sun's effect reinforces Moon's influence when Moon is dark, dark because interposed, or almost interposed, between Earth and Sun, so Sun and Moon pull from the same side. But an egg has two projections, so Sun and Moon also assist each other in producing tides when they are on opposite sides of Earth; in this case we see a full Moon.







The result? Burncoat Head in the Minas Basin, Nova Scotia, has the greatest mean range of 14.5 meters (47.5 feet) between low and high tide when Sun and Moon line up. At nearby Leaf Basin, a unique value of 16.6 meters (54.5 feet) was recorded in 1953.

High and low tides witness to the relative accelerations of portions of the ocean separated by the diameter of Earth. High tides show the "stretching" relative acceleration at different radial distances from Moon or Sun. Low tides witness to the "squeezing" relative accelerations at the same radial distance from Moon or Sun but at opposite sides of Earth.

During the time of fall, they move toward each other a distance of 1 millimeter — a thousandth of a meter, the thickness of 16 pages of this book. Why do they move toward one another? Not because of the gravitational attraction between the ball bearings; this is far too minute to bring about any "coming together." Rather, according to Newton's nonlocal view, they are both attracted toward the center of Earth. Their relative motion results from the difference in direction of Earth's gravitational pull on them, says Newton.

As another example, drop the same antique railway coach from rest in a *vertical* orientation, with the lower end of the coach initially 315 meters from the surface of Earth (Figure 2-5, right). Again release tiny ball bearings from rest at opposite ends of the coach. In this case, during the time of fall, the ball bearings move *apart* by a distance of 2 millimeters because of the greater gravitational acceleration of the one nearer Earth, as Newton would put it. This is twice the change that occurs for horizontal separation.

In either of these examples let the measuring equipment in use in the coach be just short of the sensitivity required to detect this relative motion of the ball bearings. Then, with a limited time of observation of 8 seconds, the railway coach — or, to use the earlier example, the freely falling room — serves as a free-float frame.

When the sensitivity of measuring equipment is increased, the railway coach may no longer serve as a local free-float frame unless we make additional changes. Either shorten the 20-meter domain in which observations are made, or decrease the time given to the observations. Or better, cut down some appropriate combination of space and time dimensions of the region under observation. Or as a final alternative, shoot the whole apparatus by rocket up to a region of space where one cannot detect locally the "differential gravitational acceleration" between one side of the coach and another — to use Newton's way of speaking. In another way of speaking, relative accelerations of particles in different parts of the coach must be too small to perceive. Only when these relative accelerations are too small to detect do we have a reference frame with respect to which laws of motion are simple. That's why "local" is a tricky word!

*Hold on! You just finished saying that the idea of local gravity is unnecessary. Yet here you use the "differential gravitational acceleration" to account for relative accelerations of test particles and ocean tides near Earth. Is local gravity necessary or not?*

Near Earth, two explanations of projectile paths or ocean flow give essentially the same numerical results. Newton says there is a force of gravity, to be treated like any other force in analyzing motion. Einstein says gravity differs from all other forces: Get rid of gravity locally by climbing into a free-float frame. Near the surface of Earth both explanations accurately predict relative accelerations of falling particles toward or away from one another and motions of the tides. In this chapter we use the more familiar Newtonian analysis to predict relative accelerations.

When tests of gravity are very sensitive, or when gravitational effects are large, such as near white dwarfs or neutron stars, then Einstein's predictions are not the same as Newton's. In such cases Einstein's battle-tested 1915 theory of gravity (general relativity) predicts results that are observed; Newton's theory makes incorrect predictions. This justifies Einstein's insistence on getting rid of gravity locally using free-float frames. All that remains of gravity is the relative accelerations of nearby particles — tidal accelerations.

## 2.4 REGIONS OF SPACETIME

### special relativity is limited to free-float frames

"Region of spacetime." What is the precise meaning of this term? The long narrow railway coach in Figure 2-5 probes spacetime for a limited stretch of time and in one or another single direction in space. It can be oriented north–south or east–west or up–down. Whatever its orientation, relative acceleration of the tiny ball bearings released at the two ends can be measured. For all three directions—and for all intermediate directions—let it be found by calculation that the relative drift of two test particles equals half the minimum detectable amount or less. Then throughout a cube of space 20 meters on an edge and for a lapse of time of 8 seconds (2400 million meters of light-travel time), test particles moving every which way depart from straight-line motion by undetectable amounts. In other words, the reference frame is free-float in a local region of spacetime with dimensions

$$(20 \text{ meters} \times 20 \text{ meters} \times 20 \text{ meters of space}) \times 2400 \text{ million meters of time}$$

"Region of spacetime" is four-dimensional

Notice that this "region of spacetime" is four-dimensional: three dimensions of space and one of time.

*Why pay so much attention to the small relative accelerations described above? Why not from the beginning consider as reference frames only spaceships very far from Earth, far from our Sun, and far from any other gravitating body? At these distances we need not worry at all about any relative acceleration due to a nonuniform gravitational field, and a free-float frame can be huge without worrying about relative accelerations of particles at the extremities of the frame. Why not study special relativity in these remote regions of space?*

Most of our experiments are carried out near Earth and almost all in our part of the solar system. Near Earth or Sun we cannot eliminate relative accelerations of test particles due to nonuniformity of gravitational fields. So we need to know how large a region of spacetime our experiment can occupy and still follow the simple laws that apply in free-float frames.

When is general relativity required?

For some experiments local free-float frames are not adequate. For example, a comet sweeps in from remote distances, swings close to Sun, and returns to deep space. (Consider only the head of the comet, not its 100-million-kilometer-long tail.) Particles traveling near the comet during all those years move closer together or farther apart due to tidal forces from Sun (assuming we can neglect effects of the gravitational field of the comet itself). These relative forces are called **tidal,** because similar differential forces from Sun and Moon act on the ocean on opposite sides of Earth to cause tides (Box 2-1). A frame large enough to include these particles is not free-float. So reduce spatial size until relative motion of encompassed particles is undetectable during that time. The resulting frame is very much smaller than the head of the comet! You cannot analyze the motion of a comet in a frame smaller than the comet. So instead think of a larger free-float frame that surrounds the comet for a limited time during its orbit, so that the comet passes through a series of such frames. Or think of a whole collection of free-float frames plunging radially toward Sun, through which the comet passes in sequence. In either case, motion of the comet over a small portion of its trajectory can be analyzed rigorously with respect to one of these local free-float frames using special relativity. However, questions about the entire trajectory cannot be answered using only one free-float frame; for this we require a series of frames. General relativity—the theory of gravitation—tells how to describe and predict orbits that traverse a string of adjacent free-float frames. Only general relativity can describe motion in unlimited regions of spacetime.

*Please stop beating around the bush! In defining a free-float frame, you say that every test particle at rest in such a frame remains at rest "within some specified accuracy." What accuracy? Can't you be more specific? Why do these definitions depend on whether or not we are able to perceive the tiny motion of some test particle? My eyesight gets worse. Or I take my glasses off. Does the world suddenly change, along with the standards for "inertial frame"? Surely science is more exact, more objective than that!*

Science can be "exact" only when we agree on acceptable accuracy. A 1000-ton rocket streaks 1 kilometer in 3 seconds; do you want to measure the sequence of its positions during that time with an accuracy of 10 centimeters? An astronaut in an orbiting space station releases a pencil that floats at rest in front of her; do you want to track its position to 1-millimeter accuracy for 2 hours? Each case places different demands on the inertial frame from which the observations are made. Specific figures imply specific requirements for inertial frames, requirements that must be verified by test particles. The astronaut takes off her glasses; then she can determine the position of the pencil with only 3-millimeter accuracy. Suddenly—yes!—requirements on the inertial frame have become less stringent—unless she is willing to observe the pencil over a longer period of time.

# 2.5 TEST PARTICLE

## ideal tool to probe spacetime without affecting it

Test particle defined

"Test particle." How small must a particle be to qualify as a **test particle?** It must have so little mass that, within some specified accuracy, its presence does not affect the motion of other nearby particles. In terms of Newtonian mechanics, gravitational attraction of the test particle for other particles must be negligible within the accuracy specified.

As an example, consider a particle of mass 10 kilograms. A second and less massive particle placed 10 centimeters from it and initially at rest will, in less than 3 minutes, be drawn toward it by 1 millimeter (see the exercises for this chapter). For measurements of this sensitivity or greater sensitivity, the 10-kilogram object is not a test particle. A particle counts as a test particle only when it accelerates as a result of gravitational forces without itself causing measurable gravitational acceleration in other objects—according to the Newtonian way of speaking.

Free-float frame definable because every substance falls with same acceleration

It would be impossible to define a free-float frame were it not for a remarkable feature of nature. Test particles of different size, shape, and material in the same location all fall with the same acceleration toward Earth. If this were not so, an observer inside a falling room would notice that an aluminum object and a gold object accelerate relative to one another, even when placed side by side. At least one of these test particles, initially at rest, would not remain at rest within the falling room. That is, the room would not be a free-float frame according to definition.

How sure are we that particles in the same location but of different substances all fall toward Earth with equal acceleration? John Philoponus of Alexandria argued, in 517 A.D., that when two bodies "differing greatly in weight" are released simultaneously to fall, "the difference in their time [of fall] is a very small one." According to legend Galileo dropped balls made of different materials from the Leaning Tower of Pisa in order to verify this assumption. In 1905 Baron Roland von Eötvös checked that the gravitational acceleration of wood toward Earth is equal to that of platinum within 1 part in 100 million. In the 1960s R. H. Dicke, Peter G. Roll, and Robert V. Krotkov reduced this upper limit on difference in accelerations—for aluminum and gold responding to the gravitational field of Sun—to less than 1 part in 100,000 million (less than 1 in $10^{11}$). This—and a subsequent experiment by Vladimir Braginsky and colleagues—is one of the most sensitive checks of fundamental physical principles in all of science: the equality of acceleration produced by gravity on test particles of every kind.

It follows that a particle made of any material can be used as a test particle to determine whether a given reference frame is free-float. A frame that is free-float for a test particle of one kind is free-float for test particles of all kinds.

# 2.6 LOCATING EVENTS WITH A LATTICEWORK OF CLOCKS

### only the nearest clock records an event

The fundamental concept in physics is **event.** An event is specified not only by a place but also by a time of happening. Some examples of events are emission of a particle or a flash of light (from, say, an explosion), reflection or absorption of a particle or light flash, a collision.

How can we determine the place and time at which an event occurs in a given free-float frame? Think of constructing a frame by assembling meter sticks into a cubical latticework similar to the jungle gym seen on playgrounds (Figure 2-6). At every intersection of this latticework fix a clock. These clocks are identical. They can be constructed in any manner, but their readings are in meters of light-travel time (Section 1.4).

Latticework of rods and clocks

How are the clocks to be set? We want them all to read the "same time" as one another for observers in this frame. When one clock reads midnight (00.00 hours = 0 meters), all clocks in the same frame should read midnight (zero). That is, we want the clocks to be **synchronized** in this frame.

How are the several clocks in the lattice to be synchronized? As follows: Pick one clock in the lattice as the standard and call it the **reference clock.** Start this reference

Synchronizing clocks in lattice

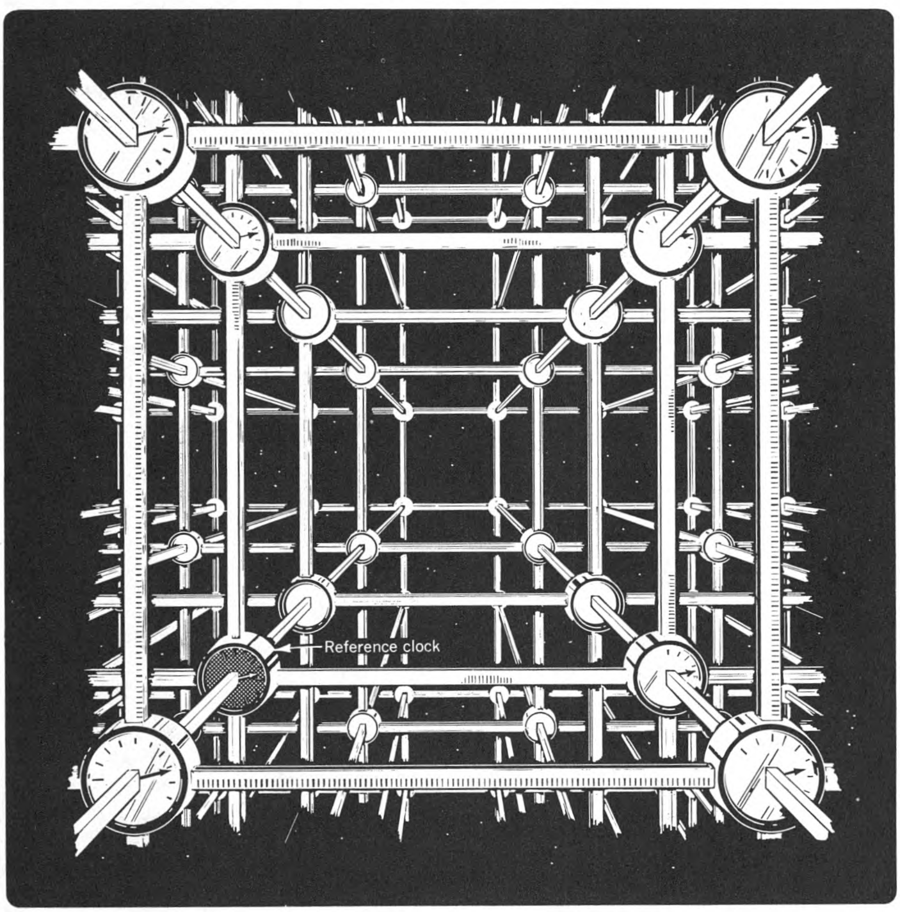



FIGURE 2-6. *Latticework of meter sticks and clocks.*

Reference event defined

clock with its pointer set initially at zero time. At this instant let it send out a flash of light that spreads out as a spherical wave in all directions. Call the flash emission the **reference event** and the spreading spherical wave the **reference flash.**

When the reference flash gets to a slave clock 5 meters away, we want that clock to read 5 meters of light-travel time. Why? Because it takes light 5 meters of light-travel time to travel the 5 meters of distance from reference clock to slave clock. So an assistant sets the slave clock to 5 meters of time long before the experiment begins, holds it at 5 meters, and releases it only when the reference flash arrives. (The assistant has zero reaction time or the slave clock is set ahead an additional time equal to the reaction time.) When assistants at all slave clocks in the lattice follow this prearranged procedure (each setting his slave clock to a time in meters equal to his own distance from the reference clock and starting it when the reference light flash arrives), the lattice clocks are said to be **synchronized.**

*This is an awkward way to synchronize lattice clocks with one another. Is there some simpler and more conventional way to carry out this synchronization?*

There are other possible ways to synchronize clocks. For example, an extra portable clock could be set to the reference clock at the origin and carried around the lattice in order to set the rest of the clocks. However, this procedure involves a moving clock. We saw in Chapter 1 that the time between two events is not necessarily the same as recorded by clocks in relative motion. The portable clock will not even agree with the reference clock when it is brought back next to it! (This idea is explored more fully in Section 4.6.) However, when we use a moving clock traveling at a speed that is a very small fraction of light speed, its reading is only slightly different from that of clocks fixed in the lattice. In this case the second method of synchronization gives a result nearly equal to the first — and standard — method. Moreover, the error can be made as small as desired by carrying the portable clock around sufficiently slowly.

Locate event with latticework

Use the latticework of synchronized clocks to determine location and time at which any given event occurs. The space position of the event is taken to be the location of the clock nearest the event. The location of this clock is measured along three lattice directions from the reference clock: northward, eastward, and upward. The time of the event is taken to be the time recorded on the same lattice clock nearest the event. The spacetime location of an event then consists of four numbers, three numbers that specify the space position of the clock nearest the event and one number that specifies the time the event occurs as recorded by that clock.

The clocks, when installed by a foresighted experimenter, will be *recording* clocks. Each clock is able to detect the occurrence of an event (collision, passage of light-flash or particle). Each reads into its memory the nature of the event, the time of the event, and the location of the clock. The memory of all clocks can then be read and analyzed, perhaps by automatic equipment.

*Why a latticework built of rods that are 1 meter long? What is special about 1 meter? Why not a lattice separation of 100 meters between recording clocks? Or 1 millimeter?*

When a clock in the 1-meter lattice records an event, we will not know whether the event so recorded is 0.4 meters to the left of the clock, for instance, or 0.2 meters to the right. The location of the event will be uncertain to some substantial fraction of a meter. The time of the event will also be uncertain with some appreciable fraction of a meter of light-travel time, because it may take that long for a light signal from the event to reach the nearest clock. However, this accuracy of a meter or less is quite adequate for observing the passage of a rocket. It is extravagantly good for measurements on planetary orbits — for a planet it would even be reasonable to increase the lattice spacing from 1 meter to hundreds of meters.

Neither 100 meters nor 1 meter is a lattice spacing suitable for studying the tracks of particles in a high-energy accelerator. There a centimeter or a millimeter would be more appropriate. The location and time of an event can be determined to whatever accuracy is desired by constructing a latticework with sufficiently small spacing.

# 2.7 OBSERVER

## ten thousand local witnesses

In relativity we often speak about the **observer.** Where is this observer? At one place, or all over the place? Answer: **The word "observer" is a shorthand way of speaking about the whole collection of recording clocks associated with one free-float frame.** No one real observer could easily do what we ask of the "ideal observer" in our analysis of relativity. So it is best to think of the observer as a person who goes around reading out the memories of all recording clocks under his control. This is the sophisticated sense in which we hereafter use the phrase "the observer measures such-and-such."

Observer defined

Location and time of each event is recorded by the clock nearest that event. We intentionally limit the observer's report on events to a summary of data collected from clocks. We do not permit the observer to report on widely separated events that he himself views by eye. The reason: travel time of light! It can take a long time for light from a distant event to reach the observer's eye. Even the order in which events are seen by eye may be wrong: Light from an event that occurred a million years ago and a million light-years distant in our frame is just entering our eyes now, after light from an event that occurred on Moon a few seconds ago. We see these two events in the "wrong order" compared with observations recorded by our far-flung latticework of recording clocks. For this reason, we limit the observer to collecting and reporting data from the recording clocks.

Observer limited to clock readings

The wise observer pays attention only to clock records. Even so, light speed still places limits on how soon he can analyze events after they occur. Suppose that events in a given experiment are widely separated from one another in interstellar space, where a single free-float frame can cover a large region of spacetime. Let remote events be recorded instantly on local clocks and transmitted by radio to the observer's central control room. This information transfer cannot take place faster than the speed of light — the same speed at which radio waves travel. Information on dispersed events is available for analysis at a central location only after light-speed transmission. This information will be full and accurate and in no need of correction — but it will be late. Thus all analysis of events must take place after — sometimes long after! — events are over as recorded in that frame. The same difficulty occurs, in principle, for a free-float frame of any size.

Nature puts an unbreakable speed limit on signals. This limit has profound consequences for decision making and control. A space probe descends onto Triton, a moon of the planet Neptune. The probe adjusts its rocket thrust to provide a slow-speed "soft" landing. This probe must carry equipment to detect its distance from Triton's surface and use this information to regulate rocket thrust on the spot, without help from Earth. Earth is never less than 242 light-minutes away from Neptune, a round-trip radio-signal time of 484 minutes — more than eight hours. Therefore the probe would crash long before probe-to-surface distance data could be sent to Earth and commands for rocket thrust returned. This time delay of information transmission does not prevent a detailed retrospective analysis on Earth of the probe's descent onto Triton — but this analysis cannot take place until at least 242 minutes

Speed limit: c
It's the law!

## SAMPLE PROBLEM 2-1

### METEOR ALERT!

Interstellar Command Center receives word by radio that a meteor has just whizzed past an outpost situated 100 light-seconds distant (a fifth of Earth-Sun distance). The report warns that the meteor is headed directly toward Command Center at one quarter light speed. Assume radio signals travel with light speed. How long do Command Center personnel have to take evasive action?

### SOLUTION

The warning radio signal and the meteor leave the outpost at the same time. The radio signal moves with light speed from outpost to Command Center, covering the 100 light-seconds of distance in 100 seconds of time. During this 100 seconds the meteor also travels toward Command Center. The meteor moves at one quarter light speed, so in 100 seconds it covers one quarter of 100 light-seconds, or 25 light-seconds of distance. Therefore, when the warning arrives at Command Center, the meteor is $100 - 25 = 75$ light-seconds away.

The meteor takes an additional 100 seconds of time to move each additional 25 light-seconds of distance. So it covers the remaining 75 light-seconds of distance in an additional time of 300 seconds.

In brief, after receiving the radio warning, Command Center personnel have a relaxed 300 seconds — or five minutes — to stroll to their meteor-proof shelter.

---

after the event. Could we gather last-minute information, make a decision, and send back control instructions? No. Nature rules out micromanagement of the far-away (Sample Problem 2-1).

# 2.8 MEASURING PARTICLE SPEED

**reference frame clocks and rods put to use**

The recording clocks reveal particle motion through the lattice: Each clock that the particle passes records the time of passage as well as the space location of this event. How can the path of the particle be described in terms of numbers? By recording locations of these events along the path. Distances between locations of successive events and time lapse between them reveal the particle speed — speed being space separation divided by time taken to traverse this separation.

Speed in meters per meter

The conventional unit of speed is meters per second. However, when time is measured in meters of light-travel time, speed is expressed in meters of distance covered per meter of time. A flash of light moves one meter of distance in one meter of light-travel time: its speed has the value unity in units of meters per meter. In contrast, a particle loping along at half light speed moves one half meter of distance per meter of time; its speed equals one half in units of meter per meter. More generally, particle speed in meters per meter is the ratio of its speed to light speed:

$$
\begin{aligned}
\text{(particle speed)} &= \frac{\text{(meters of distance covered by particle)}}{\text{(meters of time required to cover that distance)}} \\
&= \frac{\text{(particle speed in meters/second)}}{\text{(speed of light in meters/second)}}
\end{aligned}
$$

In this book we use the letter $v$ to symbolize the speed of a particle in meters of distance per meter of time, or simply meters per meter. Some authors use the lowercase Greek letter beta: $\beta$. Let $v_{conv}$ stand for velocity in conventional units (such as meters per second) and $c$ stand for light speed in the same conventional units. Then

$$v = \frac{v_{conv}}{c} \tag{2-1}$$

Test for free-float frame

From the motion of test particles through a latticework of clocks — or rather from records of coincidences of these particles with clocks — we determine whether the latticework constitutes a free-float frame. IF records show (a) that — within some specified accuracy — a test particle moves consecutively past clocks that lie in a straight line, (b) that test-particle speed calculated from the same records is constant — again, within some specified accuracy — and, (c) that the same results are true for as many test-particle paths as the most industrious observer cares to trace throughout the given region of space and time, THEN the lattice constitutes a free-float (inertial) frame throughout that region of spacetime.

*Particle speed as a fraction of light speed is certainly an unconventional unit of measure. What advantages does it have that justify the work needed to become familiar with it?*

The big advantage is that it is a measure of speed independent of units of space and time. Suppose that a particle moves with respect to Earth at half light speed. Then it travels — with respect to Earth — one half meter of distance in one meter of light travel time. It travels one half light-year of distance in a period of one year. It travels one half light-second of distance in a time of one second, one half light-minute in one minute. Units do not matter as long as we use the same units to measure distance and time; the result always equals the same number: 1/2. Another way to say this is that speed is a fraction; same units on top and bottom of the fraction cancel one another. Fundamentally, $v$ is unit-free. Of course, if we wish we can speak of "meters per meter."

# 2.9 ROCKET FRAME

## does it move? or is it the one at rest?

Rocket frame defined

Let two reference frames be two different latticeworks of meter sticks and clocks, one moving uniformly relative to the other, and in such a way that one row of clocks in each frame coincides along the direction of relative motion of the two frames (Figure 2-7). Call one of these frames **laboratory frame** and the other — moving to the right relative to the laboratory frame — **rocket frame.** The rocket is *unpowered* and coasts along with constant velocity relative to the laboratory. Let rocket and laboratory latticeworks be overlapping in the sense that a region of spacetime exists common to both frames. Test particles move through this common region of spacetime. From motion of these test particles as recorded by his own clocks, the laboratory observer verifies that his frame is free-float (inertial). From motion of the same test particles as recorded by her own clocks, the rocket observer verifies that her frame is also free-float (inertial).

Now we can describe the motion of any particle with respect to the laboratory frame. The same particles and — if they collide — the same collisions may be measured and described with respect to the free-float rocket frame as well. These particles, their paths through spacetime, and events of their collisions have an existence inde-

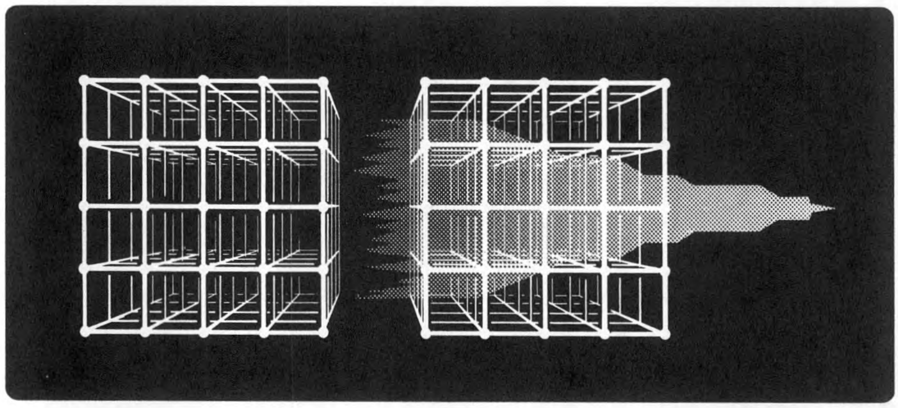


FIGURE 2-7. ***Laboratory and rocket frames.*** *A second ago the two latticeworks were intermeshed.*

Different frames lead to different descriptions

pendent of any free-float frames in which they are observed, recorded, and described. However, descriptions of these common paths and events are typically different for different free-float frames. For example, laboratory and rocket observers may not agree on the direction of motion of a given test particle (Figure 2-8). Every track that is straight as plotted with respect to one reference frame is straight also with respect to the other frame, because both are free-float frames. This straightness in both frames is possible only because *one free-float frame has uniform velocity relative to any other*

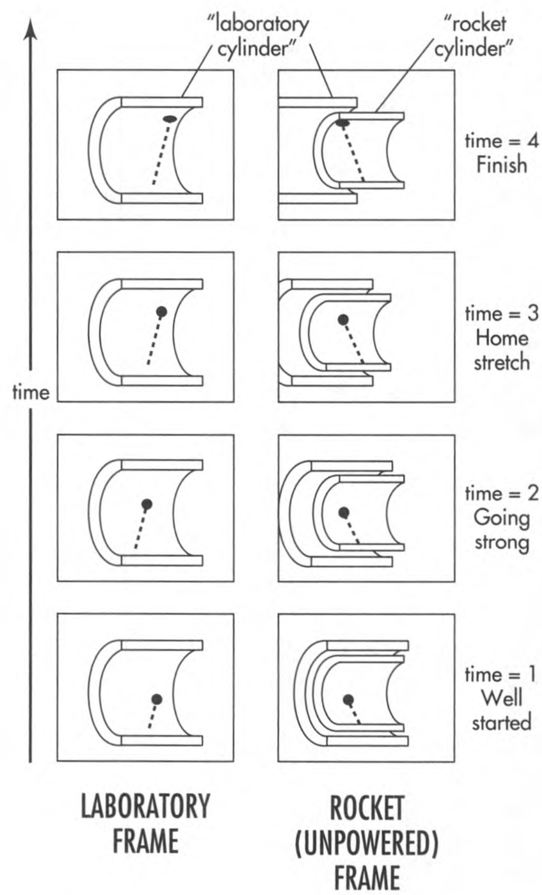



FIGURE 2-8. ***A series of "snapshots" of a typical test particle as measured from laboratory and rocket free-float frames, represented by cutaway cylinders.*** *Start at the bottom and read upward (time progresses from bottom to top).*

*overlapping free-float frame.* However, the direction of this path differs from laboratory to rocket frame, except in the special case in which the particle moves along the line of relative motion of two frames.

Many possible free-float frames

How many different free-float rocket frames can there be in a given region of spacetime? An unlimited number! Any unpowered rocket moving through that region in any direction is an acceptable free-float frame from which to make observations. More: There is nothing unique about any of these frames as long as each of them is free-float. All "rocket" frames are unpowered, all are equivalent for carrying out experiments. Even the so-called "laboratory frame" is not unique; you can rename it "Rocket Frame Six" and no one will ever know the difference! All free-float (inertial) frames are equivalent arenas in which to carry out physics experiment. That is the logical basis for special relativity, as described more fully in Chapter 3.

No unique free-float frame

*A rocket carries a firecracker. The firecracker explodes. Does this event — the explosion — take place in the rocket frame or in the laboratory frame? Which is the "home" frame for the event? A second firecracker, originally at rest in the laboratory frame, explodes. Does this second event occur in the laboratory frame or in the rocket frame?*

Events are primary, the essential stuff of Nature. Reference frames are secondary, devised by humans for locating and comparing events. A given event occurs in both frames — and in all possible frames moving in all possible directions and with all possible constant relative speeds through the region of spacetime in which the event occurs. The apparatus that "causes" the event may be at rest in one free-float frame; another apparatus that "causes" a second event may be at rest in a second free-float frame in motion relative to the first. No matter. Each event has its own unique existence. Neither is "owned" by any frame at all.

A spark jumps 1 millimeter from the antenna of Mary's passing spaceship to a pen in the pocket of John who lounges in the laboratory doorway (Section 1.2). The "apparatus" that makes the spark has parts riding in different reference frames — pen in laboratory frame, antenna in rocket frame. The spark jump — in which frame does this event occur? It is not the property of Mary, not the property of John — not the property of any other observer in the vicinity, no matter what his or her state of motion. The spark-jump event provides data for every observer.

Drive a steel surveying stake into the ground to mark the corner of a plot of land. Is this a "Daytime stake" or a "Nighttime stake"? Neither! It is just a *stake,* marking a location in *space,* the arena of surveying. Similarly an event is neither a "laboratory event" nor a "rocket event." It is just an *event,* marking a location in *spacetime,* the arena of science.

Laboratory frame or rocket frame: Which one is the "primary" free-float frame, the one "really" at rest? There is no way to tell! We apply the names "laboratory" and "rocket" to two free-float enclosures in interstellar space. Someone switches the nameplates while we sleep. When we wake up, there is no way to decide which is which. This realization leads to Einstein's Principle of Relativity and proof of the invariance of the interval, as described in Chapter 3.

# 2.10 SUMMARY

## what a free-float frame is and what it's good for

The **free-float frame** (also called the **inertial frame** and the **Lorentz frame**) provides a setting in which to carry out experiments without the presence of so-called "gravitational forces." In such a frame, a particle released from rest remains at rest and a particle in motion continues that motion without change in speed or in direction (Section 2.2), as Newton declared in his First Law of Motion.

Where does that frame of reference sit? Where do the east-west, north-south, up-down lines run? We might as well ask where on the flat landscape in the state of Iowa we see the lines that mark the boundaries of the townships. A concrete marker, to be sure, may show itself as a corner marker at a place where a north-south line meets an east-west line. Apart from such on-the-spot evidence, those lines are largely invisible. Nevertheless, they serve their purpose: They define boundaries, settle lawsuits, and fix taxes. Likewise imaginary for the most part are the clock and rod paraphernalia of the idealized inertial reference frame. Work of the imagination though they are, they provide the conceptual framework for everything that goes on in the world of particles and radiation, of masses and motions, of annihilations and creations, of fissions and fusions in every context where tidal effects of gravity are negligible.

Our ability to define a free-float frame depends on the fact that a **test particle** made of any material whatsoever experiences the same acceleration in a given gravitational field (Section 2.5).

Near a massive ("gravitating") body, we can still define a free-float frame. However, in such a frame, free test particles typically accelerate toward or away from one another because of the nonuniform field of the gravitating body (Section 2.3). This limits — in both space and time — the size of a free-float frame, the domain in which the laws of motion are simple. The frame will continue to qualify as free-float and special relativity will continue to apply, provided we reduce the spatial extent, or the time duration of our experiment, or both, until these relative, or **tidal,** motions of test particles cannot be detected in our circumscribed region of spacetime. This is what makes special relativity "special" or limited (French: *relativité restreinte:* "restricted relativity"). General relativity (the theory of gravitation) removes this limitation (Chapter 9).

So there are three central characteristics of a free-float frame. (1) We can "get rid of gravity" by climbing onto (getting into) a free-float frame. (2) The existence of a free-float frame depends on the equal acceleration of all particles at a given location in a gravitational field — in Newton's way of speaking. (3) Every free-float frame is of limited extent in spacetime. All three characteristics appear in a fuller version of the quotation by Albert Einstein that began this chapter:

> At that moment there came to me the happiest thought of my life . . . *for an observer falling freely from the roof of a house no gravitational field exists during his fall* — at least not in his immediate vicinity. That is, if the observer releases any objects, they remain in a state of rest or uniform motion relative to him, respectively, independent of their unique chemical and physical nature. Therefore the observer is entitled to interpret his state as that of "rest."

# REFERENCES

Introductory and final quotes: Excerpt from an unpublished manuscript in Einstein's handwriting, dating from about 1919. Einstein is referring to the year 1907. Italics represent material underlined in the original. Quoted by Gerald Holton in *Thematic Origins of Scientific Thought,* Revised Edition (Harvard University Press, Cambridge, Mass.,1988), page 382. Photocopy of the original provided by Professor Holton. Present translation made with the assistance of Peter von Jagow.

Figure 2-1 and Jules Verne story: Jules Verne, *A Trip From the Earth to the Moon* and *A Trip Around the Moon,* paperback edition published by Dover Publications, New York. Hardcover edition published in the Great Illustrated Classics Series by Dodd, Mead and Company, New York, 1962.

Information on Nova Scotia tides in Box 2-1: *Guinness Book of World Records 1988* (Bantam Books, New York, 1987), page 125.

Relative acceleration of different materials, Section 2-5: P. G. Roll, R. Krotkov, and R. H. Dicke, "The equivalence of inertial and passive gravitational mass," *Annals of Physics (USA),* Volume 26, pages 442–517 (1964); V. B. Braginsky and V. I. Panov, *Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki,* Volume 61, page 873 (1972) [*Soviet Physics JETP,* Volume 34, page 463 (1972)].

Experimental proofs of Einstein's general relativity theory: Clifford Will, *Was Einstein Right? Putting General Relativity to the Test* (Basic Books, New York, 1986).

# CHAPTER 2 EXERCISES

## PRACTICE

### 2-1 human cannonball

A person rides in an elevator that is shot upward out of a cannon. Think of the elevator after it leaves the cannon and is moving freely in the gravitational field of Earth. Neglect air resistance.

**a** While the elevator is still on the way up, the person inside jumps from the "floor" of the elevator. Will the person (1) fall back to the "floor" of the elevator? (2) hit the "ceiling" of the elevator? (3) do something else? If so, what?

**b** The person waits to jump until after the elevator has passed the top if its trajectory and is falling back toward Earth. Will your answers to part **a** be different in this case?

**c** How can the person riding in the elevator tell when the elevator reaches the top of its trajectory?

### 2-2 free-float bounce

Test your skill as an acrobat and contortionist! Fasten a weight-measuring bathroom scale under your feet and bounce up and down on a trampoline while reading the scale. Describe readings on the scale at different times during the bounces. During what part of each jump will the scale have zero reading? Neglecting air resistance, what is the longest part of the cycle during which you might consider yourself to be in a free-float frame?

### 2-3 practical synchronization of clocks

You are an observer in the laboratory frame stationed near a clock with spatial coordinates $x = 6$ light-seconds, $y = 8$ light-seconds, and $z = 0$ light-seconds. You wish to synchronize your clock with the one at the origin. Describe in detail and with numbers how to proceed.

### 2-4 synchronization by a traveling clock

Mr. Engelsberg does not approve of our method of synchronizing clocks by light flashes (Section 2.6).

**a** "I can synchronize my clocks in any way I choose!" he exclaims. Is he right?

Mr. Engelsberg wishes to synchronize two identical clocks, named Big Ben and Little Ben, which are relatively at rest and separated by one million kilometers, which is $10^9$ meters or approximately three times the distance between Earth and Moon. He uses a third clock, identical in construction with the first two, that travels with constant velocity between them. As his moving clock passes Big Ben, it is set to read the same time as Big Ben. When the moving clock passes Little Ben, that outpost clock is set to read the same time as the traveling clock.

**b** "Now Big Ben and Little Ben are synchronized," says Mr. Engelsberg. Is he right?

**c** How much out of synchronism are Big Ben and Little Ben as measured by a latticework of clocks — at rest relative to them both — that has been synchronized in the conventional manner using light flashes? Evaluate this lack of synchronism in milliseconds when the traveling clock that Mr. Engelsberg uses moves at 360,000 kilometers/hour, or $10^5$ meters/second.

**d** Evaluate the lack of synchronism when the traveling clock moves 100 times as fast.

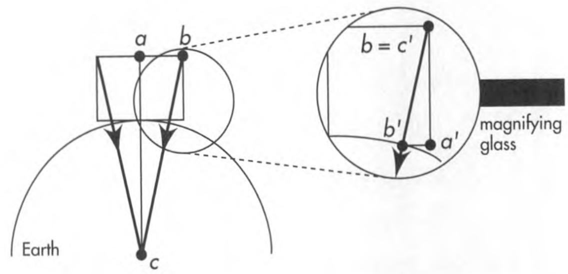
**e** Is there any earthly reason — aside from matters of personal preference — why we all should not adopt the method of synchronization used by Mr. Engelsberg?

## 2-5 Earth's surface as a free-float frame

Many experiments involving fast-moving particles and light itself are observed in earthbound laboratories. Typically these laboratories are not in free fall! Nevertheless, under many circumstances laboratories fixed to the surface of Earth can satisfy the conditions required to be called free-float frames. An example:

**a** In an earthbound laboratory, an elementary particle with speed $v = 0.96$ passes from side to side through a cubical spark chamber one meter wide. For what length of laboratory time is this particle in transit through the spark chamber? Therefore for how long a time is the experiment "in progress"? How far will a separate test particle, released from rest, fall in this time? [Distance of fall from rest $= (1/2)gt_{\text{sec}}^2$, where $g =$ acceleration of gravity $\approx 10$ meters/second$^2$ and $t_{\text{sec}}$ is the time of free fall in seconds.] Compare your answer with the diameter of an atomic nucleus (a few times $10^{-15}$ meter).

**b** How wide *can* the spark chamber be and still be considered a free-float frame for this experiment? Suppose that by using sensitive optical equipment (an **interferometer**) you can detect a test particle change of position as small as one wavelength of visible light, say 500 nanometers $= 5 \times 10^{-7}$ meter. How long will it take the test particle to fall this distance from rest? How far does the fast elementary particle of part **a** move in that time? Therefore how long can an earthbound spark chamber be and still be considered free-float for this sensitivity of detection?




**EXERCISE 2-6.** *Schematic diagram of two ball bearings falling onto Earth's surface. Not to scale.*

## 2-6 horizontal extent of free-float frame near Earth

Consider two ball bearings near the surface of Earth and originally separated horizontally by 20 meters (Section 2.3). Demonstrate that when released from rest (relative to Earth) the particles move closer together by 1 millimeter as they fall 315 meters, using the following method of similar triangles or some other method.

Each particle falls from rest toward the center of Earth, as indicated by arrows in the figure. Solve the problem using the ratio of sides of similar triangles $abc$ and $a'b'c'$. These triangles are upside down with respect to each other. However, they are similar because their respective sides are parallel: Sides $ac$ and $a'c'$ are parallel to each other, as are sides $bc$ and $b'c'$ and sides $ab$ and $a'b'$. We know the lengths of some of these sides. Side $a'c' = 315$ meters is the height of fall (greatly exaggerated in the diagram); side $ac$ is effectively equal to the radius of Earth, 6,371,000 meters. Side $ab = (1/2)$ (20 meters) equals half the original separation of the particles. Side $a'b'$ equals HALF their CHANGE in separation as they fall onto Earth's surface. Use the ratio of sides of similar triangles to find this "half-change" and therefore the entire change in separation as two particles initially 20 meters apart horizontally fall from rest 315 meters onto the surface of Earth.

## 2-7 limit on free-float frame near Earth's Moon

Release two ball bearings from rest a horizontal distance 20 meters apart near the surface of Earth's Moon. By how much does the separation between them decrease as they fall 315 meters? How many seconds elapse during this 315-meter fall? Assume that an initial vertical separation of 20 meters is increased by twice the change in horizontal separation in a fall through the same height. State clearly and completely the dimensions of the region of spacetime in which such a freely falling frame constitutes an inertial frame (to the given accuracy). Moon radius equals

1738 kilometers. Gravitational acceleration at Moon's surface: $g = 1.62$ meters/second$^2$.

## 2-8 vertical extent of free-float frame near Earth

**Note:** This exercise makes use of elementary calculus and the Newtonian theory of gravitation.

A paragraph in Section 2.3 says:

> As another example, drop the same antique [20-meter-long] railway coach from rest in a *vertical* orientation, with the lower end of the coach initially 315 meters from the surface of Earth (Figure 2-5, right). Again release two tiny ball bearings from rest at opposite ends of the coach. In this case, during the time of fall [8 seconds], the ball bearings move *apart* by a distance of two millimeters because of the greater gravitational acceleration of the one nearer Earth, as Newton would put it. This is twice the change that occurs for horizontal separation.

Demonstrate this 2-millimeter increase in separation. The following outline may be useful. Take the gravitational acceleration at the surface of Earth to be $g_o = 9.8$ meters/second$^2$ and the radius of Earth to be $r_o = 6.37 \times 10^6$ meters. More generally, the gravitational acceleration $g$ of a particle of mass $m$ a distance $r$ from the center of Earth (mass $M$) is given by the expression

$$g = \frac{F}{m} = \frac{GM}{r^2} = \frac{GM}{r_o^2}\frac{r_o^2}{r^2} = \frac{g_o r_o^2}{r^2}$$

**a** Take the differential of this equation for $g$ to obtain an approximate algebraic expression for $\Delta g$, the change in $g$, for a small change $\Delta r$ in height.

**b** Now use $\Delta y = \frac{1}{2}\Delta g t^2$ to find an algebraic expression for increase in distance $\Delta y$ between ball bearings in a fall that lasts for time $t$.

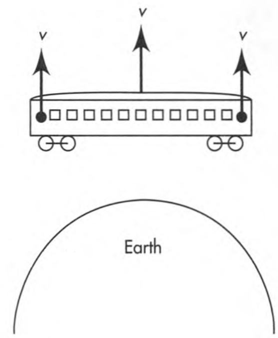
**c** Substitute numbers given in the quotation above to verify the 2-millimeter change in separation during fall.

## 2-9 the rising railway coach

You are launched upward inside a railway coach in a horizontal position with respect to the surface of Earth, as shown in the figure. After the launch, but while the coach is still rising, you release two ball bearings at opposite ends of the train and at rest with respect to the train.

**a** Riding inside the coach, will you observe the distance between the ball bearings to increase or decrease with time?

**b** Now you ride in a second railway coach launched upward in a *vertical* position with respect to




**EXERCISE 2-9.** *Free-float railway coach rising from Earth's surface, as observed in Earth frame. Two ball bearings were just released from rest with respect to the coach. What will be their subsequent motion as observed from inside the coach? Figure not to scale.*

the surface of Earth (not shown). Again you release two ball bearings at opposite ends of the coach and at rest with respect to the coach. Will you observe these ball bearings to move together or apart?

**c** In either of the cases described above, can you, the rider in the railway coach, distinguish whether the coach is rising or falling with respect to the surface of Earth solely by observing the ball bearings from inside the coach? What do you observe at the moment the coach stops rising with respect to Earth and begins to fall?

## 2-10 test particle?

**a** Verify the statement in Section 2.5 that a candidate test particle of mass 10 kilograms placed 0.1 meter from a less massive particle (initially stationary with respect to it), draws the second toward it by 1 millimeter in less than 3 minutes. If this relative motion is detectable by equipment in use at the test site, the result disqualifies the 10-kilogram particle as a "test particle." Assume that both particles are spherically symmetric. Use Newton's Law of Gravitation:

$$F = \frac{GMm}{r^2}$$

where the gravitation constant $G$ has the value $G = 6.673 \times 10^{-11}$ meter$^3$/(kilogram-second$^2$). Assume that this force does not change appreciably as the particles decrease separation by one millimeter.

**b** Section 2.3 describes two ball bearings released 20 meters apart horizontally in a freely falling railway coach. They move 1 millimeter closer together during 8 seconds of free fall, showing the limitations on this inertial frame. Verify that these ball bearings qualify as test particles by estimating the distance that one will move from rest in 8 seconds under the gravitational attraction of the other, if both were initially at rest in interstellar space far from Earth. Make your own estimate of the mass of each ball bearing.

# PROBLEMS

## 2-11 communications storm!

Sun emits a tremendous burst of particles that travels toward Earth. An astronomer on Earth sees the emission through a solar telescope and issues a warning. The astronomer knows that when the particles arrive, they will wreak havoc with broadcast radio transmission. Communications systems require three minutes to switch from broadcast to underground cable. What is the maximum speed of the particle pulse emitted by Sun such that the switch can occur in time, between warning and arrival of the pulse? Take Sun to be 500 light-seconds from Earth.

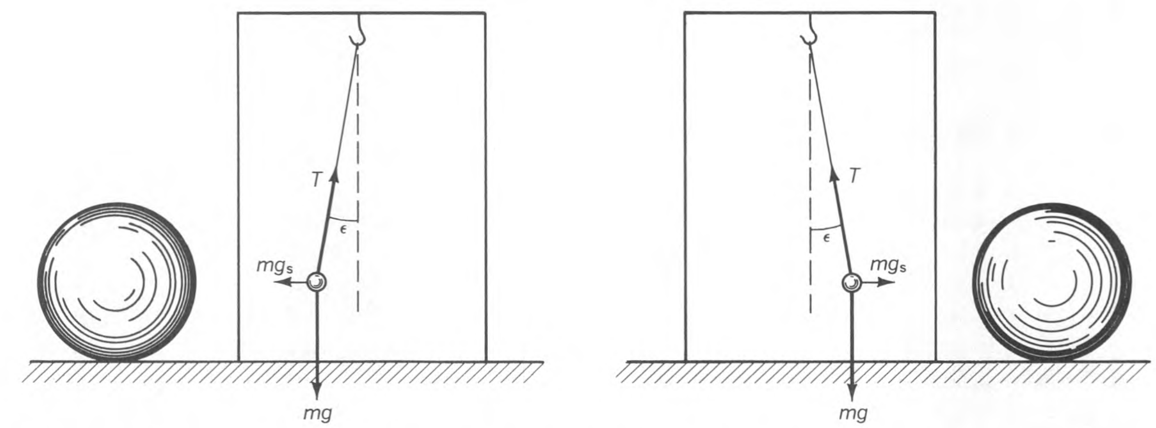
## 2-12 the Dicke experiment

**a** The Leaning Tower of Pisa is about 55 meters high. Galileo says, "The variation of speed in air between balls of gold, lead, copper, porphyry, and other heavy materials is so slight that in a fall of 100 cubits [about 46 meters] a ball of gold would surely not outstrip one of copper by as much as four fingers. Having observed this I came to the conclusion that in a medium totally devoid of resistance all bodies would fall with the same speed."

Taking four fingers to be equal to 7 centimeters, find the maximum fractional difference in the acceleration of gravity $\Delta g/g$ between balls of gold and copper that would be consistent with Galileo's experimental result.

**b** The result of the more modern Dicke experiment is that the fraction $\Delta g/g$ is not greater than $3 \times 10^{-11}$. Assume that the fraction has this more recently determined maximum value. Reckon how far behind the first ball the second one will be when the first reaches the ground if they are dropped simultaneously from the top of a 46-meter vacuum chamber. Under these same circumstances, how far would balls of different materials have to fall in a vacuum in a uniform gravitational field of 10 meters/second/second for one ball to lag behind the other one by a distance of 1 millimeter? Compare this distance with the Earth–Moon separation ($3.8 \times 10^8$ meters). Clearly the Dicke experiment was not carried out using falling balls!

**c** A plumb bob of mass $m$ hangs on the end of a long line from the ceiling of a closed room, as shown in the first figure (left). A very massive sphere at one side of the closed room exerts a horizontal gravitational force $mg_s$ on the plumb bob, where $g_s = GM/R^2$, $M$ is the mass of the large sphere, and $R$ the distance between plumb bob and the center of the sphere. This horizontal force causes a static deflection of the plumb line from the vertical by the small angle $\varepsilon$. (Similar practical example: In northern India the mass of the Himalaya Mountains results in a slight sideways deflection of plumb lines, causing difficulties in precise surveying.) The sphere is now rolled around to a corresponding position on the other side of the room (right), causing a static deflection of the plumb by an angle $\varepsilon$ of the same magnitude but in the opposite direction.




**EXERCISE 2-12, first figure.** *Left: Nearby massive sphere results in static deflection of plumb line from vertical.* ***Right:*** *Rolling the sphere to the other side results in static deflection of plumb line in the opposite direction.*

Now the angle $\varepsilon$ is very small. (Deflection due to the Himalayas is about 5 seconds of arc, which equals 0.0014 degrees.) However, as the sphere is rolled around and around outside the closed room, an observer inside the room can measure the gravitational field $g_s$ due to the sphere by measuring with greater and greater precision the total deflection angle $2\varepsilon \approx 2 \sin \varepsilon$ of the plumb line, where $\varepsilon$ is measured in radians. Derive the equation that we will need in the calculation of $g_s$.

**d** We on Earth have a large sphere effectively rolling around us once every day. It is the most massive sphere in the solar system: Sun itself! What is the value of the gravitational acceleration $g_s = GM/R^2$ due to Sun at the position of Earth? (Some constants useful in this calculation appear inside the back cover of this book.)

**e** One additional acceleration must be considered that, however, will not enter our final comparison of gravitational acceleration $g_s$ for different materials. This additional acceleration is the centrifugal acceleration due to the motion of Earth around Sun. When you round a corner in a car you are pressed against the side of the car on the outward side of the turn. This outward force—called the centrifugal pseudoforce or the centrifugal inertial force—is due to the acceleration of your reference frame (the car) toward the center of the circular turn. This centrifugal inertial force has the value $mv_{\text{conv}}^2/r$, where $v_{\text{conv}}$ is the speed of the car in conventional units and $r$ is the radius of the turn. Now Earth moves around Sun in a path that is nearly circular. Sun's gravitational force $mg_s$ acts on a plumb bob in a direction toward Sun; the centrifugal inertial force $mv_{\text{conv}}^2/R$ acts in a direction away from Sun. Compare the "centrifugal acceleration" $v_{\text{conv}}^2/R$ at the position of Earth with the oppositely directed gravitational acceleration $g_s$ calculated in part **d**. What is the net acceleration toward or away from Sun of a particle riding on Earth as observed in the (accelerated) frame of Earth?

**f** Of what use is the discussion thus far? A plumb bob hung near the surface of Earth experiences a gravitational acceleration $g_s$ toward Sun—and an equal but opposite centrifugal acceleration $mv_{\text{conv}}^2/R$ away from Sun. Therefore—in the accelerating reference frame of Earth—the bob experiences no net force at all due to the presence of Sun. Indeed this is the method by which we constructed an inertial frame in the first place (Section 2.2): Let the frame be in free fall about the center of gravitational attraction. A particle at rest on Earth's surface is in free fall about Sun and therefore experiences no net force due to Sun. What then does all this have to do with measuring the equality of gravitational acceleration for particles made of different substances—the subject of the Dicke experiment? Answer: Our purpose is to detect the difference—if any—in the gravitational acceleration $g_s$ toward Sun for different materials. The centrifugal acceleration $v^2/R$ away from Sun is presumably the same for all materials and therefore need not enter any comparison of different materials.

Consider a torsion pendulum suspended from its center by a thin quartz fiber (second figure). A light rod of length $L$ supports at its ends two bobs of equal mass made of different materials—say aluminum and gold. Suppose that the gravitational acceleration $g_1$ of the gold due to Sun is slightly greater than the acceleration $g_2$ of the aluminum due to Sun. Then there will be a slight net torque on the torsion pendulum due to Sun. For the position of Sun shown at left in the figure, show that the net torque is counterclockwise when viewed from above. Show also that the magnitude of this net torque is given by the expression

$$\begin{aligned}\text{torque} &= mg_1\, L/2 - mg_2\, L/2 = m(g_1 - g_2)\, L/2 \\ &= mg_s(\Delta g/g_s)\, L/2\end{aligned}$$

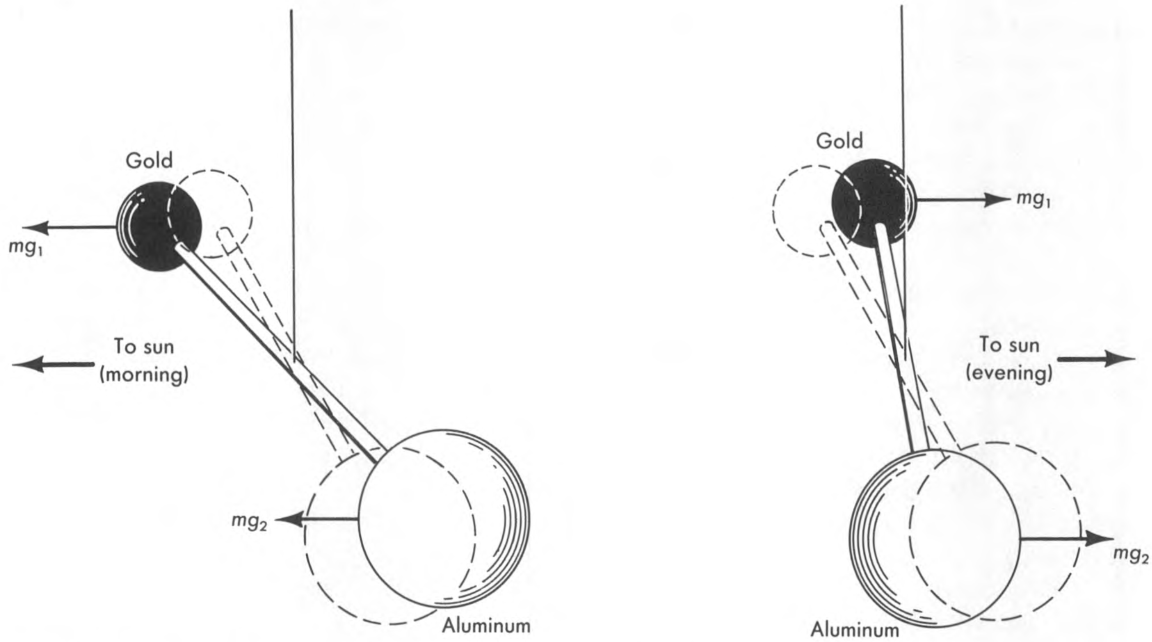
**g** Suppose that the fraction $(\Delta g/g_s)$ has the maximum value $3 \times 10^{-11}$ consistent with the results of the final experiment, that $L$ has the value 0.06 meters, and that each bob has a mass of 0.03 kilograms. What is the magnitude of the net torque? Compare this to the torque provided by the added weight of a bacterium of mass $10^{-15}$ kilogram placed on the end of a meter stick balanced at its center in the gravitational field of Earth.

**h** Sun moves around the heavens as seen from Earth. Twelve hours later Sun is located as shown at right in the second figure. Show that under these changed circumstances the net torque will have the same magnitude as that calculated in part **g** but now will be clockwise as viewed from above—in a sense opposite to that of part **g**. This change in the sense of the torque every twelve hours allows a small difference $\Delta g = g_1 - g_2$ in the acceleration of gold and aluminum to be detected using the torsion pendulum. As the torsion pendulum jiggles on its fiber because of random motion, passing trucks, Earth tremors and so forth, one needs to consider only those deflections that keep step with the changing position of Sun.

**i** A torque on the rod causes an angular rotation of the quartz fiber of $\theta$ radians given by the formula

$$\text{torque} = k\theta$$

where $k$ is called the **torsion constant** of the fiber. Show that the maximum angular rotation of the torsion pendulum from one side to the other during one




**EXERCISE 2-12, second figure.** *Schematic diagram of the Dicke experiment.* ***Left:*** *Hypothetical effect: morning.* ***Right:*** *Hypothetical effect: evening. Any difference in the gravitational acceleration of Sun for gold and aluminum should result in opposite sense of net torque on torsion pendulum in the evening compared with the morning. The large aluminum ball has the same mass as the small high-density gold ball.*

rotation of Earth is given by the expression

$$\theta_{\text{tot}} = \frac{mg_s L}{k}\left(\frac{\Delta g}{g_s}\right)$$

**i** In practice Dicke's torsion balance can be thought of as consisting of 0.030-kilogram gold and aluminum bobs mounted on the ends of a beam $6 \times 10^{-2}$ meter in length suspended in a vacuum on a quartz fiber of torsion constant $2 \times 10^{-8}$ newton meter/radian. A statistical analysis of the angular displacements of this torsion pendulum over long periods of time leads to the conclusion that the fraction $\Delta g/g$ for gold and aluminum is less than $3 \times 10^{-11}$. To what mean maximum angle of rotation from side to side during one rotation of Earth does this correspond? Random motions of the torsion pendulum—noise!—are of much greater amplitude than this; hence the need for the statistical analysis of the results.

References: R. H. Dicke, "The Eötvös Experiment," *Scientific American,* Volume 205, pages 84–94 (December, 1961). See also P. G. Roll, R. Krotkov, and R. H. Dicke, *Annals of Physics,* Volume 26, pages 442–517 (1964). The first of these articles is a popular exposition written early in the course of the Dicke experiment. The second article reports the final results of the experiment and takes on added interest because of its account of the elaborate precautions required to insure that no influence that might affect the experiment was disregarded. Galileo quote from Galileo Galilei, *Dialogues Concerning Two New Sciences,* translated by Henry Crew and Alfonso de Salvio (Northwestern University Press, Evanston, Illinois, 1950).

## 2-13 deflection of starlight by Sun

Estimate the deflection of starlight by Sun using an elementary analysis. **Discussion:** Consider first a simpler example of a similar phenomenon. An elevator car of width $L$ is released from rest near the surface of Earth. At the instant of release a flash of light is fired horizontally from one wall of the car toward the other wall. After release the elevator car is an inertial frame. Therefore the light flash crosses the car in a straight line with respect to the car. With respect to Earth, however, the flash of light is falling—because the elevator is falling. Therefore a light flash is deflected in a gravitation field, as Newton would phrase it. (How would Einstein phrase it? See Chapter 9.) As another example, a ray of starlight in its passage tangentially across Earth's surface receives a gravitational deflection (over and above any refraction by Earth's atmosphere). However, the time to cross Earth is so short, and in consequence the deflection so slight, that this effect has not yet been detected on Earth. At the surface of Sun, however, the acceleration of gravity has the much greater value of 275 meters/second/second. Moreover, the time of passage across the surface is much increased because Sun has a greater diameter, $1.4 \times 10^9$ meters. In the following, assume that the light just grazes the surface of Sun in passing.

**a** Determine an "effective time of fall" from the diameter of Sun and the speed of light. From this time of fall deduce the net velocity of fall toward Sun produced by the end of the whole period of gravitational interaction. (The maximum acceleration acting for this "effective time" produces the same net effect [calculus proof!] produced by the actual acceleration — changing in magnitude and direction along the path — in the entire passage of the ray through Sun's field of force.)

**b** Comparing the lateral velocity of the light with its forward velocity, deduce the angle of deflection. The accurate analysis of special relativity gives the same result. However, Einstein's 1915 general relativity predicted a previously neglected effect, associated with the change of lengths in a gravitational field, that produces something like a supplementary refraction of the ray of light and doubles the predicted deflection. [Deflection observed in 1947 eclipse of Sun: $(9.8 \pm 1.3) \times 10^{-6}$ radian; in the 1952 eclipse: $(8.2 \pm 0.5) \times 10^{-6}$ radian.]