CHAPTER 4

TRIP TO CANOPUS

4.1 INVITATION TO CANOPUS

is one lifetime enough?

Approximately ninety-nine light-years from Earth lies the star Canopus. The Space Agency asks us to visit it, photograph it, and return home with our records.

"But that's impossible," we object. "We have only a little over forty more years to live. We can spare at most twenty years for the outward trip, and twenty years for the return trip. Even if we could travel at the speed of light, we would need ninety-nine years merely to get there."

We are greeted with a smile and a cheery, "Think about our request a little longer, won't you?"

4.2 STRIPPED-DOWN FREE-FLOAT FRAME

throw away most clocks and rods

Troubled thoughts fill us tonight. We dream about invariance of the spacetime interval (Chapter 3). In our dream we find ourselves aboard the rocket used to establish that result (Section 3.7). However, the numbers somehow have changed from meters of distance and meters of light-travel time to light-years of distance and years of time. Suddenly we see things in a new perspective. Three revelations crowd in on us.

The flash of light that got reflected did its work—revelation number one—in establishing the identity of the spacetime interval as measured in either of the two frames. We can remember invariance of the interval and forget about the reflected flash. Eliminating it, we eliminate mirror, photodetector and, most of all, those upward-extended arrays of printout clocks in rocket and laboratory frames whose only purpose was to track the light flash.

The economy goes further. For us aboard the rocket, one reliable calendar clock is enough. As we start our trip from Earth in our dream, that clock by a happy coincidence shows noon on the Fourth of July, 2000 A.D.—and so do clocks at the Space Agency Center on Earth. We celebrate our start by setting off a firecracker.

Retain a single string of Earth-linked clocks

Later by 6 years—for us—and with a long shipboard program of research and study already completed, our rocket clock—still in our dreams—tells us it is again noon on the Fourth of July and we set off a second firecracker. At that very instant, thanks to the particular speed we had chosen for our rocket relative to Earth, we are passing Lookout Station Number 8. Lonely lighthouse, it has in it little more than a sentry person and a printout clock, one of a series that we have been passing on our trip. They have been stationed out in space, fixed one light-year apart according to Earth measurements. Each clock is calibrated and synchronized to the reference clock on Earth using a reference flash as described in Section 2.6. The laboratory latticework of Figure 2-6 has been reduced to a single rightward-stretching string of lookout stations and their clocks. That we can thus simplify our vision of what is going on from three space dimensions to one is our first revelation.

4.3 FASTER THAN LIGHT?

choose your frame. then measure velocity!

Revelation number two strikes us as—still dreaming—we pass Lookout Station Number 8, 8 light-years from Earth: What speed! We glance out of our window and see the lookout station clock print out "Fourth of July 2010 A.D."—10 years later than the Earth date of our departure. Our rocket clock reads 6 years. We are not shocked by the discrepancy in times for, apart from the change in scale from meters of light-travel time to years, the numbers are numbers we have seen before. Nor are we astonished at the identity of the spacetime interval as evaluated in the two very different frames. What amazes us is our speed. Have we actually covered a distance of 8 light-years from Earth in a time of 6 years? Can this mean we have traveled faster than light?

We have often been told that no one and no object can go faster than light. Yet here we are—in our dream—doing exactly that. Speed, yes, we suddenly say to ourselves, but speed in which frame? Ha! What inconsistency! We took the distance covered, 8 light-years, in the Earth-linked laboratory frame, but the time to cover it, 6 years, in the rocket frame!

Speed: Measure distance and time in same frame

At this point we recognize that we can talk about our speed in one reference frame or our speed in the other frame, but we get nonsense when we mix together numbers from two distinct reference frames. So we reform. First we pick for reference frame the rocket. But then we get nothing very interesting, because we did not go anywhere with respect to the rocket—we just stayed inside.

$$\begin{pmatrix}\text{our speed}\\ \text{relative to}\\ \text{rocket frame}\end{pmatrix} = \frac{\begin{pmatrix}\text{distance we cover}\\ \text{with respect to rocket}\end{pmatrix}}{\begin{pmatrix}\text{time we take to cover}\\ \text{it in rocket frame}\end{pmatrix}} = \frac{(0 \text{ light-years})}{(6 \text{ years})} = 0$$

In contrast, our speed relative to the Earth-linked reference frame, the extended laboratory, equals

$$\begin{pmatrix}\text{our speed}\\ \text{relative to}\\ \text{Earth frame}\end{pmatrix} = \frac{\begin{pmatrix}\text{distance we cover}\\ \text{with respect to Earth}\end{pmatrix}}{\begin{pmatrix}\text{time we take to cover}\\ \text{it in Earth frame}\end{pmatrix}} = \frac{(8\text{ light-years})}{(10\text{ years})} = 0.8\text{ light-speed}$$

In other words we—and the rocket—travel, relative to Earth, at 80 percent of the maximum possible speed, the speed of light. Revelation number two is our discovery that speed in the abstract makes no sense, that speed has meaning only when referred to a clearly stated frame of reference. Relative to such a frame we can approach arbitrarily close to light speed but never reach it.

4.4 ALL OF SPACE IS OURS!

in one lifetime: go anywhere in the cosmos

Revelation number three strikes us as—dreaming on—we think more about passing Earth-linked lookout stations. Moving at 80 percent of light speed, we travel 8 light-years in the Earth-linked frame in 6 years of our rocket time. Continuing at the same rate will get us to Canopus in 74 years of our rocket time. Better than 99 years, but not good enough.

Let's use—in imagination—a faster rocket! We suddenly remember the super-rocket discussed in demonstrating the invariance of the spacetime interval (Section 3.8). Converting meters of distance and time to years, we realize that traveling in the super-rocket would bring us to Earth-linked Lookout Station Number 20, *20* Earth-frame light-years from Earth, in 6 years of our rocket time. When passing this station, we can see that this station clock reads 20.88 years. Therefore in the Earth-linked frame our super-rocket speed amounts to $20/20.88 = 0.958$ light speed. Continuing at the same speed would bring us to Canopus in 29.7 years of our rocket time. This is nearly short enough to meet our goal of 20 years.

Revelation number three gives us a dizzying new sense of freedom. By going fast enough we can get to Canopus in five minutes of our rocket time if we want! In fact, no matter how far away an object lies, and no matter how short the time allotted to us, nothing in principle stops us from covering the required distance in that time. We have only to be quite careful in explaining this new-found freedom to our Space Agency friends. Yes, we can go any distance the agency requires, however great, provided they specify the distance in the *Earth-linked* reference frame. Yes, we can make it in any nonzero time the agency specifies, however short, provided they agree to measure time on the *rocket* clock we carry along with us.

Five minutes to Canopus—or to any star!

To be sure, the Earth-linked system of lookout stations and printout clocks will record us as traveling at less than the speed of light. Lookouts will ultimately complain to the Space Agency how infernally long we take to make the trip. But when our Space Agency friends quiz the lookouts a bit more, they will have to confess the truth: When they look through our window as we shoot by station after station, they can see that our clock reads much less than theirs, and in terms of our own rocket clock we are meeting the promised time for the trip.

Our dream ends with sunlight streaming through the bedroom window. We lie there savoring the three revelations: economy of description of two events in a reference frame stripped down to one space dimension, speed defined always with respect to a specified reference frame and thus never exceeding light speed, and freedom to go arbitrarily far in a lifetime.

4.5 FLIGHT PLAN

out and back in 40 years to meet our remote descendents

Wide awake now, we face yesterday's question: Shall we go to Canopus, 99 light-years distant, as the Space Agency asks? Yes. And yes, we shall live to return and report.

We take paper and pencil and sketch our plan. The numbers have to be different from those we dreamed about. Trial and error gives us the following plan: After a preliminary run to get up to speed, we will zoom past Earth at $99/101 = 0.9802$ light speed. We will continue at that speed all the 99 light-years to Canopus. We will make a loop around it and record in those few minutes, by high-speed camera, the features of that strange star. We will then return at unaltered speed, flashing by our finish line without any letup, and as we do so, we will toss out our bundle of records to colleagues on Earth. Then we will slow down, turn, and descend quietly to Earth, our mission completed.

Round trip: 202 Earth years

The first long run takes 101 Earth years. We have already decided to travel at a speed of 99/101, or 99 light-years of distance in 101 years of time. Going at that speed for 101 Earth years, we will just cover the 99 light-years to Canopus. The return trip will likewise take 101 Earth years. Thus we will deliver our records to Earth 202 Earth-clock years after the start of our trip.

Round trip: 40 astronaut years

Even briefer will be the account of our trip as it will be perceived in the free-float rocket frame. Relative to the ship we will not go anywhere, either on the outbound or on the return trip. But time will go on ticking away on our shipboard clock. Moreover our biological clock, by which we age, and all other good clocks carried along will tick away in concord with it. How much time will that rocket clock rack up on the outbound trip? Twenty years. How do we know? We reach this answer in three steps. First, we already know from records in the Earth-linked laboratory frame that the spacetime interval — the proper time — between departure from Earth and arrival at Canopus will equal 20 years:

$$\begin{aligned}
& \qquad\qquad\quad \textbf{Laboratory} \qquad\qquad\quad \textbf{Laboratory} \\
(\text{interval})^2 &= (\text{time separation})^2 - (\text{space separation})^2 \\
&= (101 \text{ years})^2 - (99 \text{ years})^2 \\
&= 10{,}201 \text{ years}^2 - 9801 \text{ years}^2 \\
&= 400 \text{ years}^2 = (20 \text{ years})^2
\end{aligned}$$

Second, as the saying goes, "interval is interval is interval": The spacetime interval is invariant between frames. The interval as registered in the rocket frame must therefore also have this 20-year value. Third, in the rocket frame, separation between the two events (departure from Earth and arrival at Canopus) lies all in the time dimension, zero in the space dimension, since we do not leave the rocket. Therefore separation in rocket time itself between these two events is the proper time and must likewise be 20 years:

$$\begin{aligned}
& \qquad\qquad\quad \textbf{Rocket} \qquad\qquad\qquad \textbf{Rocket} \\
(\text{interval})^2 &= (\text{time separation})^2 - (\text{space separation})^2 \\
&= (\text{time separation})^2 - (\text{zero})^2 \\
&= (\text{rocket time})^2 = (\text{proper time})^2 \\
&= (20 \text{ years})^2
\end{aligned}$$

We boil down our flight plan to bare bones and take it to the Space Agency for approval: Speed 99/101 = 0.9802 light speed; distance 99 light-years out, 99 light-years back; time of return to Earth 202 years after start; astronaut's aging during trip, 40 years. The responsible people greet the plan with enthusiasm. They thank us for volunteering for a mission so unprecedented. They ask us to take our proposal before the Board of Directors for final approval. We agree, not realizing what a hornets' nest we are walking into.

The Board of Directors consists of people from various walks of life, set up by Congress to assure that major projects have support of the public at large. The media have reported widely on our proposal in the weeks before we meet with the board, and many people with strong objections to relativity have written to voice their opinions. A few have met with board members and talked to them at length. We are unaware of this as we enter the paneled board room.

At the request of the chairman we summarize our plan. The majority appear to welcome it. Several of their colleagues, however, object.

4.6 TWIN PARADOX

a kink in the path explains the difference

"Your whole plan depends on relativity," stresses James Fastlane, "but relativity is a swindle. You can see for yourself that it is self-contradictory. It says that the laws of physics are identical in all free-float frames. Very well, here's your rocket frame and here's Earth frame. You tell me that identical clocks, started near Earth at identical times, each in one of these free-float frames, will read very different time lapses. You go away and return only 40 years older, while we and our descendants age 202 years. But if there's any justice, if relativity makes any sense at all, it should be equally possible to regard *you* as the stay-at-home. Relative to you, *we* speed away in the opposite direction and return. Hence we should be younger than you when we meet again. In contrast, you say you will be younger than we are. This is a flat contradiction. Nothing could show more conclusively that neither result can be right. Aging is aging. It is impossible to live long enough to cover a distance of 99 light-years twice — going and coming. Forget the whole idea."

Which twin travels?

"Jim," we reply, "your description is the basis for the famous Twin Paradox, in which one twin stays on Earth while the other takes the kind of round trip we have been describing. Which twin is older when they come together again? I would like to leave this question for a minute and consider a similar trip across the United States.

"We all know, Jim, that every July you drive straight north on Interstate Highway 35 from Laredo, Texas, on the Mexican border, to Duluth, Minnesota, near the Canadian border. Your tires roll along a length of roadway equal to 2000 kilometers and the odometer on your car shows it.

"I too drive from Laredo to Duluth, but last year I had to make a stop in Cincinnati, Ohio, on the way. I drove northeast as straight as I could from Laredo to Cincinnati, 1400 kilometers, and northwest as straight as I could from Cincinnati to Duluth, another 1400 kilometers. Altogether, my tires rolled out 2800 kilometers. When we left Laredo you could have said that my route was deviating from yours, and I could have said with equal justice that yours was deviating from mine. The great difference between our travels is this, that my course has a sharp turn in it. That's why my kilometerage is greater than yours in the ratio of 2800 to 2000."

Curved path in space is a longer path

Fastlane interrupts: "Are you telling me that the turn in the rocket trajectory at Canopus explains the *smaller* aging of the rocket traveler? The turn in your trip to Duluth made your travel distance *longer*, not shorter."

"That is the difference between path length in Euclidean space geometry and wristwatch time in Lorentz spacetime geometry," we reply. "In Euclidean geometry the *shortest path length* between two points is achieved by the traveler who does not change direction. All indirect paths are longer than this minimum. In spacetime the *greatest aging* between two events is experienced by the traveler who does not change direction. For all travelers who change direction, the total proper time, the total wristwatch time, the total aging is *less* than this maximum.

Astronaut who turns around ages less . . .

"The distinction between distance in Euclidean geometry and aging in spacetime comes directly from the contrast between *plus* sign in the expression for distance between two locations and *minus* sign in the expression for interval between two events. In going to Duluth by way of Cincinnati I use the *plus* sign:

$$\begin{pmatrix}\text{distance:}\\ \text{Laredo to}\\ \text{Cincinnati}\end{pmatrix}^2 = \begin{pmatrix}\text{northward}\\ \text{separation:}\\ \text{Laredo to}\\ \text{Cincinnati}\end{pmatrix}^2 + \begin{pmatrix}\text{eastward}\\ \text{separation:}\\ \text{Laredo to}\\ \text{Cincinnati}\end{pmatrix}^2$$

"Contrast this with motion in spacetime. In analyzing my trip to Canopus, I use the minus sign:

. . . because of a minus sign!

$$\begin{pmatrix}\text{proper time:}\\ \text{Earth to}\\ \text{Canopus}\end{pmatrix}^2 = \begin{pmatrix}\text{rocket time:}\\ \text{Earth to}\\ \text{Canopus}\end{pmatrix}^2 = \begin{pmatrix}\text{Earth time:}\\ \text{Earth to}\\ \text{Canopus}\end{pmatrix}^2 - \begin{pmatrix}\text{Earth distance:}\\ \text{Earth to}\\ \text{Canopus}\end{pmatrix}^2$$

"The contrast between a plus sign and a minus sign: This is the distinction between distance covered during travel in space and time elapsed—aging—during travel in spacetime."

4.7 LORENTZ CONTRACTION

go a shorter distance in a shorter time

As James Fastlane ponders this response, Dr. Joanne Short breaks in. "The Twin Paradox is not the only one you have to explain in order to convince us of the correctness of your analysis. Look at the outward trip as observed by you yourself, the rocket traveler. You reach Canopus after just 20 years of your time. Yet we know that Canopus lies 99 light-years distant. How can you possibly cover 99 light-years in 20 years?"

"That is exactly what I dreamed about, Joanne!" we reply. "First of all, it is confusing to combine distances measured in one reference frame with time measured in another reference frame. The 99-light-year distance to Canopus is measured with respect to the Earth-linked frame, while the 20 years recorded on the outward traveler's clock refers to the rocket frame. No wonder the result appears to imply a rate of travel faster than light. Why not take what I paid for fuel for *my* car last week and divide it by the number of gallons you bought today for *your* car, to figure the cost of a gallon of fuel? A crazy, mixed-up, wrong way to work out cost—but no crazier than that way to figure speed!

"But your question about time brings up a similar question about distance: distance between Earth and Canopus measured in the frame in which they are at rest does not agree with the distance between them measured from a rocket that moves along the line connecting them.

Canopus much closer for astronaut

"Any free-float frame is as good as any other for analyzing motion — that is the Principle of Relativity! So think of the entire outward trip in terms of rocket measurements. At the starting gun (or firecracker) Earth is rushing past the rocket at speed 99/101. Twenty years later Canopus arrives at the rocket, Canopus also traveling at that speed, 99/101 in that rocket frame. This means that for the rocket traveler the Earth-Canopus distance is only about 20 light-years. In fact it is just the fraction (99/101) of 20 light-years, so that at speed 99/101 this distance is covered in exactly 20 years."

"Of course. We are dealing with **Lorentz contraction,**" huffs Professor Bright, who thinks any objection to relativity is a waste of time. He has no head for politics, so does not appreciate how important it is for the public to accept the expenditures proposed for this project.

Lorentz contraction

He continues, "Think of a very long stick lying with one end at Earth, the other end at Canopus. Each observer, with the help of colleagues, measures the position of the two ends of this stick *at the same time* in his or her frame. By this means the outward rocket traveler measures a shorter length of the stick — a smaller Earth – Canopus distance — than does an observer in the Earth-linked frame in which the stick lies at rest.

"The factor by which the stick appears contracted in the rocket frame is just the same as the ratio of rocket time to Earth time for the outward trip. This ratio is (20 years)/(101 years). Hence the rocket observer measures the Earth – Canopus distance to be (99 light-years)(20/101) = 19.6 light-years — just a bit less than 20 light-years, as you said.

"Everybody has a satisfactory picture: The astronaut can get to Canopus in 20 years of rocket time because the astronaut's measurements show Canopus to be slightly less than 20 light-years distant. We on Earth agree that the time lapse on the rocket clock is 20 years, but our 'explanation' rests on the invariance of the interval between the events of departure from Earth and arrival at Canopus." Professor Bright pounds the table: "Why are you giving this poor astronaut such a hard time, when relativity is so utterly simple?" He is surprised by the outburst of laughter from other board members and the audience in the room.

4.8 TIME TRAVELER

visit the future. don't come back.

Laura Long has been thoughtfully following the argument. She comments, "You know, we have been discussing you as a space traveler. But you are a *time* traveler as well. Do you realize that by traveling to Canopus and back at 99/101 of light speed, you journey six generations forward in time: 202 years at 33 years per generation? So you will be able to visit your great-great-great-great-great-grandchildren at a cost of only 40 years of your life."

"Yes, I did think of that," we reply. "Time and space are not so different in this respect. Just as we can travel to as great an Earth-linked distance as we want in as short a rocket time as we want, so we can also travel as far forward into Earth's future as we wish.

Travel to Earth's future

"While I was trying various numbers in making up the proposed plan, I realized that if we traveled not at 99/101 light speed but at 9999/10,001 light speed, then a round trip would take not 40 rocket years but only 3.96 rocket years and 198 Earth years. Ten such round trips will age us 39.6 years and bring us back finally at an Earth time about two thousand years in the future, or some year in the fortieth century. That is not six generations ahead, but sixty generations, an additional time equal to one third of recorded history on Earth."

"Why stop there?" pursues Laura Long excitedly. "Why not go even faster, make more round trips, and learn the ultimate fate of Earth and its solar system—or even the still more remote future of the Universe as a whole? Then you could report back to us whether the Universe expands forever or ends in a crunch."

"Sorry, but no report back to our century is possible," smiles Professor Bright. "There are differences between travel in time and travel in space. To begin with, we can stand still on Earth if we choose and go nowhere in space with respect to that frame. Concerning travel through time, however, we have no such choice! Even when we stand stock still on Earth, we nevertheless travel gently but inevitably forward in time. Time proceeds inexorably!

Time travel is one way

"Second, time travel is one way. You may be able to buy a round-trip ticket to Canopus, but you can get only a one-way ticket to the fortieth century. You can't go backward in time. Time won't reverse."

Turning to us he adds, "As for the fate of the solar system and the end of the Universe, our descendants may meet you there as fellow observers, but we ourselves will have to bid you a firm and final 'good-bye' as you leave us on any of the trips we have been discussing. The French *au revoir*—until we meet again—will not do."

4.9 RELATIVITY OF SIMULTANEITY

we turn around; our changing colleagues say Earth's clock flies forward

By this time James Fastlane has gotten his second wind. "I am still stuck in this Twin Paradox thing. The time for the outward trip is less as measured in the rocket frame than as measured in the Earth frame. But if relativity is correct, every free-float frame is equivalent. As you sit on the rocket, you feel yourself to be at rest, stationary, motionless; you measure our Earth watch-station clocks to be zipping by you at high velocity. Who cares about labels? For you these Earth clocks are in motion! Therefore the time for the outward trip should be less as measured on the ('moving') Earth clock than as measured on your ('stationary') rocket clock."

Rocket observer: Fewer Earth-clock ticks on outward trip . . .

We nod assent and he continues. "Nothing prevents us from supposing the existence of a series of rocket lookout stations moving along in step with your rocket and strung out at separations of one light-year as measured in your rocket frame, all with clocks synchronized in your rocket frame and running at the same rate as your rocket clock. Now, as Earth passes each of these rocket lookout stations in turn, won't those stations read and record the times on the passing Earth clock to be less than their own times? Otherwise how can relativity be correct?"

"Yes, your prediction is reasonable," we reply.

. . . also fewer Earth-clock ticks on return trip

"And on the return trip will not the same be true: Returning-rocket lookout stations will measure and record time lapses on the passing Earth clock to be less than on their own clocks?"

"That conclusion is inevitable if relativity is consistent."

"Aha!" exclaims Mr. Fastlane, "Now I've got you! If Earth clock is measured by rocket lookout stations to show smaller time lapses during the outward trip—and also during the return trip—then obviously total Earth time must be less than rocket round-trip time. But you claim just the opposite: that total rocket time is less than Earth time. This is a fundamental contradiction. Your relativity is wrong!" Folding his arms he glowers at us.

There is a long silence. Everyone looks at us except Professor Bright, who has his head down. It is hard to think with all this attention. Yet our mind runs over the trip again. Going out . . . coming back . . . turning around . . . that's it!

"All of us have been thinking the wrong way!" we exclaim. "We have been talking as if there is only a single rocket frame. True, the same vehicle, with its traveler, goes out and returns. True, a single clock makes the round trip with the traveler. But this vehicle *turns around*—reverses its direction of travel—and that changes everything.

"Maybe it's simpler to think of two rockets, each moving without change of velocity. We ride on the first rocket going out and on the second rocket coming back. Each of these two is really a rocket *frame:* each has its own long train of lookout stations with recording clocks synchronized to its reference clock (Figure 4-1). The traveler can be thought of as 'jumping trains' at Canopus—from outward-bound rocket frame to inward-bound rocket frame—carrying the calendar clock.

Astronaut jumps from outgoing frame to returning frame

"Now follow Mr. Fastlane's prescription to analyze the trip in the rocket frame, but with this change: make this analysis using *two* rocket frames—one outward bound, the other inward bound.

"It is 20 years by outward-rocket time when the traveler arrives at Canopus. That is the reading on all lookout station clocks in that outward-rocket frame. One of those lookout stations is passing Earth when this rocket time arrives. Its clock, synchronized to the clock of the outward traveler at Canopus, also reads 20 years. What time does that rocket lookout-station guard read on the passing Earth clock? For the rocket observer Earth clock reads less time by the same factor that rocket clocks read less time (20 years at arrival at Canopus) for Earth observers (who read 101 years on their own clocks). This factor is 20/101. Hence for the outward-rocket observer the Earth clock must read 20/101 times 20 years, or 3.96 years."

Outgoing rocket:
As it arrives at Canopus,
Earth clock reads 3.96 years

"What!" explodes Fastlane. "According to your plan, the turnaround at Canopus occurs at 101 years of Earth time. Now you say this time equals less than 4 years on Earth clock."

"No sir, I do *not* say that," we reply, feeling confident at last. "I did say that *at the same time* as the outgoing rocket arrives at Canopus, Earth clock reads 3.96 years *as measured in that outgoing rocket frame.* An equally true statement is that *at the same time* as the outgoing rocket arrives at Canopus, Earth clock reads 101 years *as measured in the Earthbound frame.* Apparently observers in different reference frames in relative motion do not agree on what events occur *at the same time* when these events occur far apart along the line of relative motion."

Once again Professor Bright supplies the label. "Yes, that is called **relativity of simultaneity.** Events that occur at the same time—simultaneously—judged from

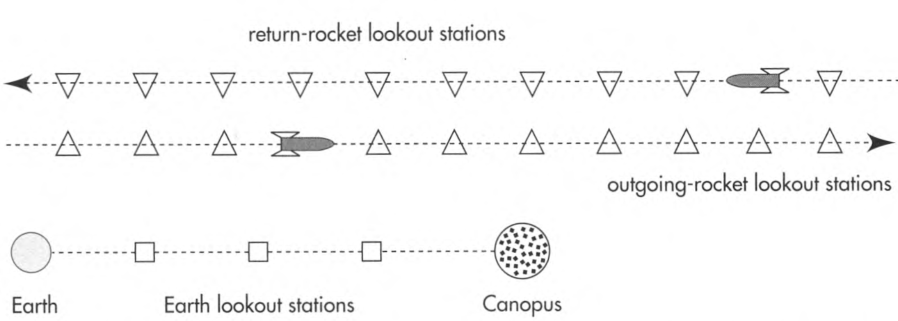

FIGURE 4-1. ***Schematic plot in the Earth-linked frame showing the outgoing rocket and the return rocket used in the round trip between Earth and Canopus.*** *The two rockets meet at Canopus, where the traveler jumps from outgoing rocket to return rocket. Each reference frame has its own string of lookout stations, at rest and synchronized in that frame, shown by small squares, triangles, and inverted triangles. In this figure the outgoing and return rocket lines of motion are displaced vertically for purposes of analysis; in reality, all motion lies along the single line between Earth and Canopus. The figure is* not *to scale!*

one free-float frame but far apart along the line of relative motion do not occur simultaneously as judged from another free-float frame.

"As an example of relativity of simultaneity, consider either chain of lookout stations strung along the line of relative motion. If all clocks in the lookout stations of one frame strike exactly at noon in that frame, these strikes are not simultaneous as measured in another frame in relative motion with respect to the first. This is called **relative synchronization of clocks.**

"Incidentally, most of the so-called 'paradoxes' of relativity, one of which we are considering now, turn on misconceptions about relativity of simultaneity."

Dr. Short breaks in. "What about the returning rocket? What time on the Earth clock will the returning rocket lookout station measure as the traveler starts back?"

Returning rocket: As it leaves Canopus, Earth clock reads 198.04 years

"That shouldn't be too difficult to figure out," we reply. "We know that the clock on the returning rocket reads 40 years when we arrive home on Earth. And the Earth clock reads 202 years on that return. Both of these readings occur at the same place (Earth), so we do not need to worry about relativity of simultaneity of that reading. And during the return trip Earth clock records less elapsed time than rocket clocks' 20 years by the same factor, 20/101, or a total elapsed time of $20 \times 20/101 = 3.96$ years according to return rocket observations. Therefore at the earlier turnaround, return rocket observers will see Earth clock reading $202 - 3.96 = 198.04$ years."

"Wait a minute!" bellows Fastlane. "First you say that the rocket observer sees the Earth clock reading 3.96 years at turnaround in the outward-bound frame. Now you say that the rocket observer sees the Earth clock read 198.04 years at turnaround in the inward-bound frame. Which one is right?"

"Both are right," we reply. "The two observations are made from two different frames. Each of these frames has a duly synchronized system of lookout-station clocks, as does the Earth-linked frame (Figure 4-1). The so-called Twin Paradox is resolved by noticing that between the Earth-clock reading of 3.96 years, taken from the outward rocket lookout station at turnaround and the Earth-clock reading of 198.04 years, taken by the returning-rocket lookout station at turnaround, there is a difference of 194.08 years.

Forward "jump" in Earth clock results from frame change

"This 'jump' appears on no single clock but is the result of the traveler changing frames at Canopus. Yet this jump, or difference, resolves the paradox: For the traveler, the Earth clock reads small time lapses on the outward leg—and also small time lapses on the return leg—but it jumps *way* ahead at turnaround. This jump accounts for the large value of Earth-aging during the trip: 202 years. In contrast the traveler ages only 40 years during the trip (Table 4-1).

"And notice that the traveler is unique in the experience of changing frames; only the traveler suffers the terrible jolt of reversing direction of motion. In contrast, the

TABLE 4-1

OBSERVATIONS OF EVENTS ON CANOPUS TRIP

Event	Time measured in Earth-linked frame	Time measured by traveler	*Earth-clock reading observed by* outgoing-rocket lookout stations passing Earth	return-rocket lookout stations passing Earth
Depart Earth	0 years	0 years	0 years	
Arrive Canopus	101 years	20 years	20 years $\times$ 20/101 = 3.96 years	
Depart Canopus	101 years	20 years	3.96 years	202 − 3.96 = 198.04 years
Arrive Earth	202 years	40 years		202 years

Earth observer stays relaxed and comfortable in the same frame during the astronaut's entire trip. Therefore there is no symmetry between rocket traveler and Earth dweller, so no genuine contradiction in their differing time lapses, and the story of the twins is not a paradox.

"In fact, the observer in each of the three frames — Earth-linked, outward-rocket, and inward-rocket — has a perfectly consistent and nonparadoxical interpretation of the sequence of events. However, in accounting for disagreements between his or her readings and those of observers in other free-float frames, each observer infers some misbehavior of measuring devices in these other frames. Each observes less elapsed time on clocks in the other frame than on his or her own clocks (time stretching or time dilation). Each thinks that an object lying along the line of relative motion and at rest in another frame is contracted (Lorentz contraction). Each thinks that lookout-station clocks in other frames are not synchronized with one another (relative synchronization of clocks). As a result, each cannot agree with other observers as to which events far apart along the line of relative motion occur at the same time (relativity of simultaneity)."

All observers agree on result, disagree on reason

"Boy," growls Fastlane, "all these different reference frames sure do complicate the story!"

"Exactly!" we exclaim. "These complications arise because observations from any one frame are limited and parochial. All disagreements can be bypassed by talking only in the invariant language of spacetime interval, proper time, wristwatch time. The proper time from takeoff from Earth to arrival at Canopus equals 20 years, period. The proper time from turnaround at Canopus to rearrival at Earth equals 20 years, period. The sum equals 40 years as experienced by the astronaut, period. On the Earth clock, the proper time between departure and return is 202 years, period. End of story. Observers in all free-float frames reckon proper times — spacetime intervals between these events — using their differing space and time measurements. However, once the data are translated into the common language of proper time, every observer agrees. Proper times provide a universal language independent of reference frame."

Spacetime interval is universal language

4.10 EXPERIMENTAL EVIDENCE

objects large and small, slow and fast: many witnesses for the Canopus trip

Alfred Missouri has remained silent up to this point. Now he declares, "All this theory is too much for me. I won't believe a word you say unless you can show me an experimental demonstration."

We reply, "Atomic clocks have been placed on commercial airliners and carried around Earth, some in an eastward direction, others in a westward direction. In each case the airliner clocks were compared with reference clocks at the U.S. Naval Observatory before and after their trips. These clocks disagreed. Results were consistent with the velocity-related predictions of special relativity.

"Airliner" test of twin effect

"This verification of special relativity has two minor difficulties and a major one. Minor difficulties: (1) Each leg of a commercial airliner's trip may be at a different speed, not always accurately known and for which the time-stretching effect must be separately calculated. Also, temperature and pressure effects on airborne clocks are hard to control in a commercial airliner. (2) More fundamentally, Earth rotates, carrying the reference Naval Observatory clocks eastward around the center of Earth. Earth center can be regarded as the inertial point in free-float around Sun. With

BOX 4-1

DO WE NEED GENERAL RELATIVITY? NO!

The group takes a break and mills around the conference room, chatting and eating refreshments. Joanne Short approaches us juggling coffee, a donut, and her notes.

"I didn't want to embarrass you in public," she says, "but isn't your plan faulty because of the turnaround? You can't be serious about leaping from one high-speed rocket to another rocket going in the opposite direction. That means certain death! Be realistic: You and your rocket will have to slow down over some time period, come to rest at Canopus, then speed up again, this time headed back toward Earth. During this change of velocity you will be thrown against the front of the rocket ship, as I'm thrown when I slam on my car brakes. Release a test particle from rest and it will hurtle forward! Surely you are not in an inertial (free-float) frame. Therefore you cannot use special relativity in your analysis of this time period. What does that do to your description of the 'jump ahead' of Earth clocks as you slow down and speed up again? Don't you need general relativity to analyze events in accelerated reference frames?"

"Oh yes, general relativity can describe events in the accelerated frame," we reply, "but so can special relativity if we take it in easy steps! I like to think of a freight yard with trains moving at different speeds along parallel tracks. Each train has its own string of recording clocks along its length, each string synchronized in that particular train frame. Each adjacent train is moving at a slightly different speed from the one next to it. Now we can change frames by walking *across* the trains, stepping from the top of one freight car to the top of the freight car rolling next to it at a slightly different speed.

"Let these trains become rocket trains in space. Each train then has an observer passing Earth as we step on that train. Each observer, by prearrangement, reads the Earth clock *at the same time* that we step onto his train ('at the same time' as recorded in *that* frame). When you assemble all these data later on, you find that the set of observers on the sequence of trains see the Earth clock jumping forward in time much faster than would be expected. The net result is similar to the single horrible jerk as you jump from the outgoing rocket to the incoming rocket.

"Notice that it takes a whole set of clocks in different frames, all reading the single Earth clock, to establish this result. So there is never any contradiction between a single clock in one frame and a single clock in any other frame. In this case special relativity can do the job just fine."

The directors reassemble and Joanne Short, smiling, takes her place with them.

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respect to this center, one airborne clock moves even faster eastward than Earth's surface, while the other one — heading west with respect to the surface — with respect to Earth's center also moves eastward, but more slowly. Taking account of these various relative velocities adds further complication to analysis of results.

"We overcome these two minor difficulties by having an airplane fly round and round in circles in the vicinity of a single ground-based reference atomic clock.

Then — to a high accuracy — only *relative* motion of these two clocks enters into the special-relativity analysis.

"On November 22, 1975, a U.S. Navy P3C antisubmarine patrol plane flew back and forth for 15 hours at an altitude of 25,000 to 35,000 feet (7600 to 10,700 meters) over Chesapeake Bay in an experiment arranged by Carroll Alley and collaborators. The plane carried atomic clocks that were compared by laser pulse with identical clocks on the ground. Traveling at an average speed of 270 knots (140 meters per second), the airborne clocks lost an average of 5.6 nanoseconds $= 5.6 \times 10^{-9}$ seconds due to velocity-related effects in the 15-hour flight. The expected special-relativity difference in clock readings for this relative speed is 5.7 nanoseconds. This result is remarkably accurate, considering the low relative velocity of the two clocks: 4.7×10^{-7} light speed.

"Circling airplane" test of twin effect

"The *major* difficulty with all of these experiments is this: A high-flying airplane is significantly farther from Earth's center than is the ground-based clock. Think of an observer in a helicopter reading the clocks of passing airplanes and signaling these readings for comparison to a ground-based clock directly below. These two clocks — the helicopter clock and the Earthbound clock — are at rest with respect to one another. Are they in the same inertial (free-float) frame? The answer is No.

Trouble: Large frame is not inertial

"We know that a single inertial reference frame near Earth cannot extend far in a vertical direction (Section 2.3). Even if the two clocks — helicopter and Earthbound — were dropped in free fall, they could not both be in the same inertial frame. Released from rest 30,000 feet one above the other, they would increase this relative distance by 1 millimeter in only 0.3 second of free fall — too rapid a change to be ignored. But the experiment went on not for 0.3 second but for 15 hours!

"Since the helicopter clock and Earthbound clock are not in the same inertial frame, their behavior cannot be analyzed by special relativity. Instead we must use general relativity — the theory of gravitation. General relativity predicts that during the 15-hour flight the higher-altitude clock in the Chesapeake Bay experiment will record *greater* elapsed time by 52.8 nanoseconds due to the slightly reduced gravitational field at altitudes at which the plane flew. From this must be subtracted the 5.7 nanoseconds by which the airborne clock is predicted to record *less* elapsed time due to effects of relative velocity. These velocity effects are predicted by both special relativity and general relativity and were the only results quoted above. The overall predicted result equals $52.8 - 5.7 = 47.1$ nanoseconds net gain by the high-altitude clock compared with the clock on the ground. Contrast this with the measured value of 47.2 nanoseconds.

Solution: Use general relativity

"Hence for airplanes flying at conventional speeds and conventional altitudes, tidal-gravitational effects on clocks can be greater than velocity-dependent effects to which special relativity is limited. In fact, the Chesapeake Bay experiment was conducted to verify the results of general relativity: The airplane pilot was instructed to fly as slowly as possible to reduce velocity effects! The P3C patrol plane is likely to stall below 200 knots, so a speed of 270 knots was chosen.

"In all these experiments the time-stretching effect is small because the speed of an airplane is small compared to the speed of light, but atomic clocks are now so accurate that these speed effects are routinely taken into account when such clocks are brought together for direct comparison."

Professor Bright chimes in. "What the astronaut says is correct: We do not have large clocks moving fast on Earth. On the other hand, we have a great many small clocks moving very fast indeed. When particles collide in high-speed accelerators, radioactive fragments emerge that decay into other particles after an average lifetime that is well known when measured in the rest frame of the particle. When the radioactive particle moves at high speed in the laboratory, its average lifetime is significantly longer as measured on laboratory clocks than when the particle is at rest. The amount of lengthening of this lifetime is easily calculated from the particle speed in the same way the astronaut calculates time stretching on the way to and from

"High-speed radioactive particle" test of twin effect

Canopus. The time-stretch factor can be as great as 10 for some of these particles: the fast-moving particles are measured to live 10 times longer, on average, than their measured lifetime when at rest! The experimental results agree with these calculations in all cases we have tried. Such time stretching is part of the everyday experience of high-energy particle physicists.

Earth frame: Free-float for particle experiments

"And for these increased-lifetime experiments there is no problem of principle in making observations in an inertial, free-float frame. While they are decaying, particles cover at most a few tens of meters of space. Think of the flight of each particle as a separate experiment. An individual experiment lasts as long as it takes one high-speed particle to move through the apparatus—a few tens of meters of light-travel time. Ten meters of light-travel time equals about 33 nanoseconds, or 33×10^{-9} seconds.

"Can we construct an inertial frame for such happenings? Two ball bearings released from rest say 20 meters apart do not move together very far in 33 nanoseconds! Therefore these increased-lifetime experiments could be done, in principle, in free-float frames. It follows that special relativity suffices to describe the behavior of the 'radioactive-decay clocks' employed in these experiments. We do not need the theory of gravitation provided by general relativity.

"Oscillating iron nucleus" test of twin effect

"Of course, in none of these high-speed particle experiments do particles move back and forth the way our astronaut friend proposes to do between Earth and Canopus. Even that back-and-forth result has been verified for certain radioactive iron nuclei vibrating with thermal agitation in a solid sample of iron. Atoms in a hotter sample vibrate back and forth faster, on average, and thus stay younger, on average, than atoms in a cooler sample. In this case the 'tick of the clock' carried by an iron atom is the period of electromagnetic radiation ('gamma ray') given off when its nucleus makes the transition from a radioactive state to one that is not radioactive. For detailed reasons that we need not go into here, this particular 'clock' can be read with very high accuracy. Beyond all such details, the experimental outcome is simply stated: Clocks that take one or many round trips at higher speed record a smaller elapsed time than clocks that take one or many round trips at lower speed.

Twin effect verified!

"These various results—plus many others we have not described—combine to give overwhelming experimental support for the predictions of the astronaut concerning the proposed trip to Canopus."

Dr. Bright sits back in his chair with a smile, obviously believing that he has disposed of all objections single-handedly.

"Yes," we conclude, "about the reality of the effect there is no question. Therefore if you all approve, and the Space Agency provides that new and very fast rocket, we can be on our way."

The meeting votes approval and our little story ends.

REFERENCES

The "airliner check" of time stretching (Section 4.10) is reported in J. C. Hafele and Richard E. Keating, *Science,* Volume **177,** pages 166–167 and 168–170 (14 July 1972).

The "patrol plane" check of general relativity (Section 4.10) is reported by Carroll O. Alley in *Quantum Optics, Experimental Gravity, and Measurement Theory,* edited by Pierre Meystre and Marlan O. Scully (Plenum, New York, 1983). See also 1976 physics Ph.D. theses by Robert A. Reisse and Ralph E. Williams, University of Maryland.

The "radioactive nuclei" check of time stretching (Section 4.10) is reported in R. V. Pound and G. A. Rebka, Jr., *Physical Review Letters,* Volume **4,** pages 274–275 (1960).

CHAPTER 4 EXERCISES

Note: The following exercises are related to the story line of this chapter. Additional exercises may be selected from Chapter 3 or the Special Topic on the Lorentz Transformation following Chapter 3.

4-1 practical space travel

In 2200 A.D. the fastest available interstellar rocket moves at $v = 0.75$ of the speed of light. James Abbott is sent in this rocket at full speed to Sirius, the Dog Star (the brightest star in the heavens as seen from Earth), a distance $D = 8.7$ light-years as measured in the Earth frame. James stays there for a time $T = 7$ years as recorded on his clock and then returns to Earth with the same speed $v = 0.75$. Assume Sirius is at rest relative to Earth. Let the departure from Earth be the reference event (the zero of time and space for all observers).

According to Earth-linked observers:

a At what time does the rocket arrive at Sirius?

b At what time does the rocket leave Sirius?

c At what time does the rocket arrive back at Earth?

According to James's observations:

d At what time does he arrive at Sirius?

e At what time does he leave Sirius?

f At what time does he arrive back at Earth?

g As he moves toward Sirius, James is accompanied by a string of *outgoing* lookout stations along his direction of motion, each one with a clock synchronized to his own. What is the spatial distance between Earth and Sirius, according to observations made with this outgoing string of lookout stations?

h One of James's outgoing lookout stations, call it Q, passes Earth at the same time (in James's outgoing frame) that James reaches Sirius. What time does Q's clock read at this event of passing? What time does the clock on Earth read at this same event?

i As he moves back toward Earth, James is accompanied by a string of *incoming* lookout stations along his direction of motion, each one with a clock synchronized to his own. One of these incoming lookout stations, call it Z, passes Earth at the same time (in James's incoming frame) that James leaves Sirius to return home. What time does Z's clock read at this event of passing? What time does the clock on Earth read at this same event?

To *really* understand the contents of Chapter 4, repeat this exercise many times with new values of v, D, and T that you choose yourself.

4-2 one-way twin paradox?

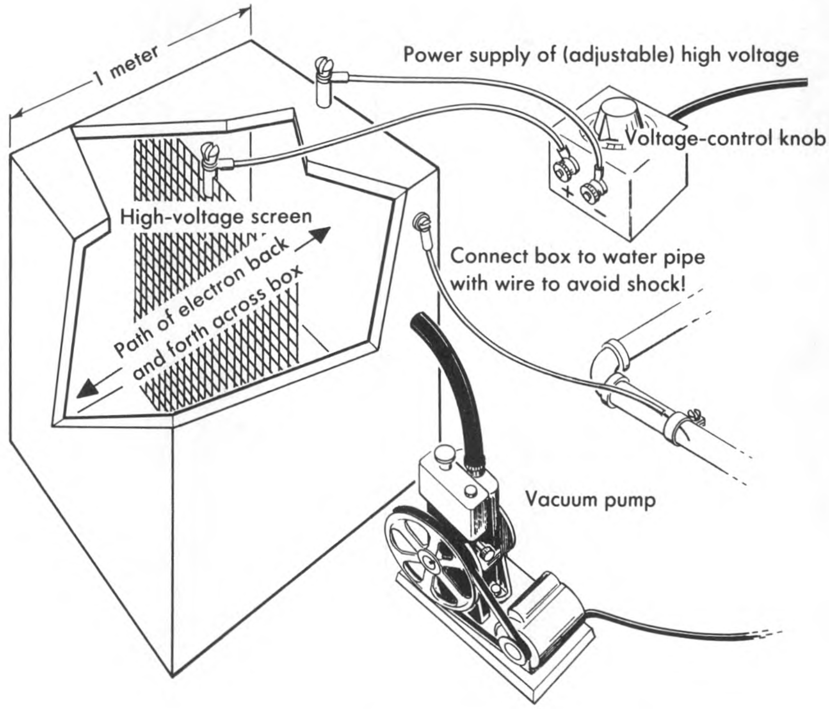
A worried student writes, "I still cannot believe your solution to the Twin Paradox. During the outward trip to Canopus, each twin can regard the other as moving away from him; so how can we say which twin is younger? The answer is that the twin in the rocket makes a turn, and in Lorentz spacetime geometry, the greatest aging is experienced by the person who does not turn. This argument is extremely unsatisfying. It forces me to ask: What if the rocket breaks down when I get to Canopus, so that I stop there but cannot turn around? Does this mean that it is no longer possible to say that I have aged less than my Earthbound twin? But if not, then I would never have gotten to Canopus alive." Write a half-page response to this student, answering the questions politely and decisively.

4-3 a relativistic oscillator

In order to test the laws of relativity, an engineer decides to construct an oscillator with a very light oscillating bob that can move back and forth very fast. The lightest bob known with a mass greater than zero is the electron. The engineer uses a cubical metal box, whose edge measures one meter, that is warmed slightly so that a few electrons "boil off" from its surfaces (see the figure). A vacuum pump removes air from the box so that electrons may move freely inside without colliding with air molecules. Across the middle of the box—and electrically insulated from it—is a metal screen charged to a high positive voltage by a power supply. A voltage-control knob on the power supply can be turned to change the DC voltage V_o between box and screen. Let an electron boiled off from the inner wall of the box have very small velocity initially (assume that the initial velocity is zero). The electron is attracted to the positive screen, increases speed toward the screen, passes through a hole in the screen, slows down as it moves away from the attracting screen, stops just short of the opposite wall of the box, is pulled back toward the screen; and in this way oscillates back and forth between the walls of the box.

a In how short a time T can the electron be made to oscillate back and forth on one round trip between the walls? The engineer who designed the equipment claims that by turning the voltage control knob high enough he can obtain as high a frequency of oscillation $f = 1/T$ as desired. Is he right?

b For sufficiently low voltages the electron will be nonrelativistic—and one can use Newtonian me-



EXERCISE 4-3. *Relativistic oscillator with electron as oscillating bob.*

chanics to analyze its motion. For this case the frequency of oscillation of the electron is increased by what factor when the voltage on the screen is doubled? **Discussion:** At corresponding points of the electron's path before and after voltage doubling, how does the Newtonian kinetic energy of the electron compare in the two cases? How does its velocity compare in the two cases?

c What is a definite formula for frequency f as a function of voltage in the nonrelativistic case? Wait as late as possible to substitute numbers for mass of electron, charge of electron, and so forth.

d What is the frequency in the extreme relativistic case in which over most of its course the electron is moving . . . (rest of sentence suppressed!) . . . ? Call this frequency f_{max}.

e On the same graph, plot two curves of the dimensionless quantity f/f_{max} as functions of the dimensionless quantity $qV_o/(2mc^2)$, where q is the charge on the electron and m is its mass. First curve: the nonrelativistic curve from part **c** to be drawn heavily in the region where it is reliable and indicated by dashes elsewhere. Second curve: the extreme relativistic value from part **d**, also with dashed lines where not reliable. From the resulting graph estimate quantitatively the voltage of transition from the nonrelativistic to the relativistic region. If possible give a simple argument explaining why your result does or does not make sense as regards order of magnitude (that is, overlooking factors of 2, π, etc.).

f Now think of the round-trip "proper period" of oscillation τ experienced by the electron and logged by its recording wristwatch as it moves back and forth across the box. At low electron speeds how does this proper period compare with the laboratory period recorded by the engineer? What happens at higher electron speeds? At extreme relativistic speeds? How is this reflected in the "proper frequency" of oscillation f_{proper} experienced by the electron? On the graph of part **e** draw a rough curve in a different color or shading showing qualitatively the dimensionless quantity f_{proper}/f_{max} as a function of $qV_o/(2mc^2)$.